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Representation of fuzzy subsets by Galois connections. (English) Zbl 1454.06002
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Summary: There is a great deal of fuzziness in our everyday natural language, and thus fuzzy subsets have come to represent a direct generalisation of the indicator function of a classical subset. On the other hand, a Galois connection is given by two opposite order-inverting maps whose composition yields two closure operations between ordered sets. We present the one-to-one correspondence between a set of all fuzzy subsets and a set of all Galois connections. The essential correspondences are built with the help of α -cuts, which represent fuzzy subsets by means of classical sets. Moreover, we present a relationship between strong fuzzy negations in the lattices and Galois connections. The various extensions of fuzzy subsets from the point of view of nestedness and negations are recalled. Other fruitful properties and connections with related studies are included.

MSC:

06A15 Galois correspondences, closure operators (in relation to ordered sets)
03E72 Theory of fuzzy sets, etc.
06B23 Complete lattices, completions
03B65 Logic of natural languages

Cited in **2** Documents

Keywords:

Galois connections; algebra; fuzzy subsets; hedges; nestedness

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