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Galois connections between lattices of preradicals induced by ring epimorphisms. (English)

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The authors introduced the concept of a Galois connection between the lattices of preradicals R -pr and S -pr of two rings R and S induced by any adjoint pair of functors between the categories in an earlier paper [the authors, Appl. Categ. Struct. 24, No. 3, 241–268 (2016; Zbl 1345.16030)]. The paper under review continues this study. For the Galois connection $\langle \phi, \psi \rangle$ induced by the adjoint pair $\langle F, G \rangle$ of functors, several results concerning preradicals are found. Let R and S be associative rings with identity and let τ be any preradical over R and σ any preradical over S . Then the pretorsion-free class associated with $\phi(\tau)$ and the pretorsion class associated with $\psi(\sigma)$ are described explicitly. It is also shown that ϕ preserves idempotency and ψ preserves radicals.

If $f : A \rightarrow B$ is a ring homomorphism, then two pairs of adjoint functors $\langle F, G \rangle$ and $\langle G, H \rangle$ exist with induced Galois connections $\langle \phi, \psi \rangle$ and $\langle \zeta, \xi \rangle$ respectively. If τ is a preradical on A and N is a B -submodule, then $\psi(\tau)(N)$ is shown to be the least B -submodule containing $\tau(A)N$ and $\xi(\tau)(N)$ is shown to be the greatest B -submodule of N that is contained in $\tau(A)N$. This leads to a description of the pretorsion and pretorsion-free classes that correspond to the induced Galois connections.

Whereas the earlier paper of the authors [loc. cit.] focused on the case where S is a quotient ring, a slightly more general setting is considered here, viz. the case where $f : A \rightarrow B$ is a ring epimorphism. In this case, it is shown that $\psi(\sigma)$ is the greatest and $\zeta(\sigma)$ is the least extension of any preradical σ on B and it follows that ϕ and ξ are surjective and ψ and ζ are injective. The class of all extensions of any preradical σ on A is characterized. If τ is an idempotent preradical (resp. radical) on B , then it is proved that the interval $[\zeta(\tau), \psi(\tau)]$ is closed under products (resp. coproducts). Characterizations are also found for the equalities $\phi = \xi$ and $\psi = \zeta$ to be true.

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MSC:

- 16S90 Torsion theories; radicals on module categories (associative algebraic aspects)
- 06A15 Galois correspondences, closure operators (in relation to ordered sets)
- 18A40 Adjoint functors (universal constructions, reflective subcategories, Kan extensions, etc.)
- 16D40 Free, projective, and flat modules and ideals in associative algebras

Keywords:

Galois connections; ring epimorphisms; preradicals; adjoint functors

Full Text: DOI

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