

**Chiaselotti, Giampiero; Infusino, F.**

**Some classes of abstract simplicial complexes motivated by module theory.** (English)

Zbl 1452.13027

J. Pure Appl. Algebra 225, No. 1, Article ID 106471, 44 p. (2021).

Authors' abstract: In this paper we analyze some classes of abstract simplicial complexes relying on algebraic models arising from module theory. To this regard, we consider a leftmodule on a unitary ring and find models of abstract complexes and related set operators having specific regularity properties, which are strictly interrelated to the algebraic properties of both the module and the ring. Next, taking inspiration from the aforementioned models, we carry out our analysis from modules to arbitrary sets. In such a more general perspective, we start with an abstract simplicial complex and an associated set operator. Endowing such a set operator with the corresponding properties obtained in our module instances, we investigate in detail and prove several properties of three subclasses of abstract complexes. More specifically, we provide uniformity conditions in relation to the cardinality of the maximal members of such classes. By means of the notion of OSS-bijection, we prove a correspondence theorem between a subclass of closure operators and one of the aforementioned families of abstract complexes, which is similar to the classic correspondence theorem between closure operators and Moore systems. Next, we show an extension property of a binary relation induced by set systems when they belong to one of the above families. Finally, we provide a representation result in terms of pairings between sets for one of the three classes of abstract simplicial complexes studied in this work.

Reviewer: Amir Mafi (Sanandaj and Tehran)

#### MSC:

- 13F55 Commutative rings defined by monomial ideals; Stanley-Reisner face rings; simplicial complexes
- 18D15 Closed categories (closed monoidal and Cartesian closed categories, etc.)
- 18D20 Enriched categories (over closed or monoidal categories)
- 06F20 Ordered abelian groups, Riesz groups, ordered linear spaces
- 06A15 Galois correspondences, closure operators (in relation to ordered sets)
- 08A02 Relational systems, laws of composition
- 16D10 General module theory in associative algebras

Cited in 2 Documents

#### Keywords:

abstract simplicial complexes; closure systems; left-modules; set operators; pairings

Full Text: DOI

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