Oh, Ju-Mok; **Kim, Yong Chan**
The properties of residuated connections and Alexandrov topologies. (English)
Zbl 1450.54005
Korean J. Math. 28, No. 2, 311-321 (2020).

Given partially ordered sets (posets for short) S and T, a pair (g, f) of maps $g : S \to T$ and $f : T \to S$ is called a *Galois connection* or an *adjunction* between S and T provided that $f(t) \leqslant s$ if and only if $t \leqslant g(s)$ for every $t \in T$ and every $s \in S$ (see, e.g., [*G. Gierz* et al., Continuous lattices and domains. Cambridge: Cambridge University Press (2003; Zbl 1088.06001)] for more detail). Moreover, given a set X, a map $a : X \times X \to [0, +\infty]$ is called a *metric* provided that the following two properties hold: (1) $a(x, z) \leqslant a(x, y) + a(y, z)$ for every $x, y, z \in X$; (2) $a(x, x) = 0$ for every $x \in X$ (see, e.g., [*D. Hofmann* (ed.) et al., Monoidal topology. A categorical approach to order, metric, and topology. Cambridge: Cambridge University Press (2014; Zbl 1297.18001)] for more detail and notice that the definition skips some properties of the classical metrics of, e.g., [*R. Engelking*, General topology. Rev. and compl. ed. Berlin: Heldermann Verlag (1989; Zbl 0684.54001)]). The pair (X, a) is then called a *metric space*. Lastly, there exists the concept of *Alexandroff topology*, i.e., a topology, in which arbitrary (and not just finite) intersections of open sets are open. In particular, given a poset $(S, \leqslant)$, the classical Alexandroff topology on the set S is generated by the down-sets $\downarrow s = \{s' \in S \,|\, s' \leqslant s\}$ for $s \in S$ (see, e.g., the above-mentioned "Monoidal Topology" of Hofmann et al. for more detail).

The present paper replaces posets S and T in the above with metric spaces (X, a) and (Y, b) respectively, introducing thus the concept of residuated connection in the following way. Given metric spaces (X, a) and (Y, b), and maps $f : X \to Y$ and $g : Y \to X$, the quadruple (a, f, g, b) is called a *residuated connection* provided that $b(f(x), y) = a(x, g(y))$ for every $x \in X$ and every $y \in Y$. The paper then considers a suitable lattice-valued analogue of Alexandroff topology, and studies relationships between lattice-valued Alexandroff topologies and metrics (in the above sense). More precisely, the authors construct a lattice-valued Alexandroff topology from a metric and vice versa.

The paper is well written, contains most of its required preliminaries, is quite easy to read, and will be of interest to all those researchers, who study lattice-valued topology or partial orders. Its main deficiency is total lack of discussions of the obtained results and somewhat obscured motivation of the authors in general (for example, almost the whole introductory section deals with the concept of rough set of [*Z. Pawlak*, Int. J. Comput. Inform. Sci. 11, 341–356 (1982; Zbl 0501.68053)], which is then never used in the rest of the paper). Additionally, the condition defining Alexandroff topology at the very beginning of Theorem 3.7 on page 315 is not clear.

Reviewer: Sergejs Solovjovs (Praha)

MSC:

54A40 Fuzzy topology
06A15 Galois correspondences, closure operators (in relation to ordered sets)
06A75 Generalizations of ordered sets
54E35 Metric spaces, metrizability
54F05 Linearly ordered topological spaces, generalized ordered spaces, and partially ordered spaces

Keywords:

Alexandroff topology; complete residuated lattice; lattice-valued topology; non-symmetric pseudo-metric; residuated connection; rough set

Full Text: DOI

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