

Santocanale, Luigi; Wehrung, Friedrich**The equational theory of the weak Bruhat order on finite symmetric groups.** (English)

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The weak Bruhat order on the symmetric group S_n is a lattice $P(n)$ called the permutohedron on n letters. The Tamari lattice $A(n)$ is a lattice retract of $P(n)$. Given a poset E , there is an ‘extended permutahedron’ lattice defined from E denoted $R(E)$.

The main results of this important paper are:

- A. The class of all $P(n)$ has a decidable equational theory.
- B. The class of all $A(n)$ has a decidable equational theory.
- C. There is a lattice identity that holds in all $P(n)$ but fails in some 3,338-element lattice.
- D. Any finite meet-semidistributive lattice embeds in $R(E)$ for some countable poset E . (The poset E for this embedding can always be taken to be the directed union of finite dismantlable lattices.) In particular, the class of all $R(E)$ generates the variety of all lattices.

Reviewer: [Keith Kearnes \(Boulder\)](#)**MSC:**

- 06B20 Varieties of lattices
- 03C85 Second- and higher-order model theory
- 06A07 Combinatorics of partially ordered sets
- 06A15 Galois correspondences, closure operators (in relation to ordered sets)
- 06B10 Lattice ideals, congruence relations
- 06B25 Free lattices, projective lattices, word problems
- 20B30 Symmetric groups

Cited in 1 Document

Keywords:

lattice; identity; weak order; permutohedron; Cambrian lattice; Tamari lattice; monadic second-order logic; decidability; score; bounded homomorphic image; subdirectly irreducible; splitting lattice; splitting identity; polarized measure; sub-tensor product; box product; dismantlable lattice

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