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Valuations: bi, tri, and tetra. (English) Zbl 1450.03002
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This paper aims to present a systematic overview on valuational presentations of consequence relations, and the Galois connections between spaces of valuations and spaces of consequence relations. Galois connections between sets of valuations and sets of arguments are introduced and their associated closure operations. Sets of arguments and sets of valuations are characterized that are closed with regard to this closure operations. Finally the effects of adding connectives to the language are considered. A sound but partial complete sequent calculus is presented.

From the conclusion of the paper we quote: “Attending to the Galois connections between sets of arguments and sets of valuations has proved useful in a range of applications to logics that are reflexive, monotonic, and completely transitive, which can all be handled through the lens of bivaluations. In this paper, it is shown how to extend this toolkit to logics that might fail to be reflexive or completely transitive or both, by adding up to two more values to the valuations. Finally, the behaviour of connectives witnessing the bilattice structure of the resulting tetravaluations is considered.”

Reviewer: [Albert Hoogewijs \(Gent\)](#)

MSC:

- [03B05](#) Classical propositional logic
- [03B22](#) Abstract deductive systems
- [03B50](#) Many-valued logic
- [06A15](#) Galois correspondences, closure operators (in relation to ordered sets)

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Keywords:

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