Castro, L. P.; **Pesetskaya, E.**

A composite material with inextensible-membrane-type interface. (English) Zbl 1447.74013

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Summary: We consider a model of a composite material with "inextensible-membrane-type" interface conditions. An analytic solution of a stationary heat conduction problem in an unbounded doubly periodic two-dimensional composite whose matrix and inclusions consist of isotropic temperature-dependent materials is given. In the case where the conductive properties of the inclusions are proportional to those of the matrix, the problem is transformed into a fully linear boundary value problem for doubly periodic analytic functions. The solution makes it possible to calculate the average properties over the unit cell and discuss the effective conductivity of the composite. We present numerical examples to indicate some peculiarities of the solution.

MSC:

74E30 Composite and mixture properties
74F05 Thermal effects in solid mechanics
74Q15 Effective constitutive equations in solid mechanics
80A19 Diffusive and convective heat and mass transfer, heat flow

Keywords:

two-dimensional doubly periodic material; effective heat conductivity; non-ideal contact condition; analytical solution

Full Text: DOI

References:

1 Amosov, AA, Panasenko, GP. The problem of thermo-chemical formation of a composite material: Properties of solutions and homogenization. J Math Sci 2012; 181(5): 541-577. (Translation from Probl Mat Anal 63: 3-33.) · Zbl 1252.35261

2 Andrianov, IV, Bolshakov, VI, Danishevs'kyy, VV. Asymptotic study of imperfect interfaces in conduction through a granular composite material. Proc R Soc Lond Ser A Math Phys Eng Sci 2010; 466: 2707-2725. · Zbl 1211.74056

3 Berlyand, L, Mityushev, V. Generalized Clausius-Mossotti formula for random composite with circular fibers. J Stat Phys 2001; 102(1-2): 115-145. · Zbl 1072.82586

4 Casado-Daz, J. Some smoothness results for the optimal design of a two-composite material which minimizes the energy. Calc Var Part Differ Eq 2015; 53(3-4): 649-673. · Zbl 1317.49045

5 Castro, LP, Pesetskaya, E. A transmission problem with imperfect contact for an unbounded multiply connected domain. Math Meth Appl Sci 2010; 33(4): 517-526. · Zbl 1187.30035

6 Castro, LP, Pesetskaya, E, Rogosin, S. Effective conductivity of a composite material with non-ideal contact conditions. Compl Var Ellipt Eq 2009; 54(12): 1085-1100. · Zbl 1184.30029

7 Dalla Riva, M, Musolino, P. A singularly perturbed non-ideal transmission problem and application to the effective conductivity of a periodic composite. SIAM J Appl Math 2013; 73(1): 24-46. · Zbl 1412.74064

8 El Jarroudi, M . Homogenization of a nonlinear elastic fibre-reinforced composite: A second gradient nonlinear elastic material. J Math Anal Appl 2013; 403(2): 487-505. · Zbl 1430.74126

9 Gryshchuk, SV. Effective conductivity of 2D ellipse-elliptical ring composite material. Zb Pr Inst Mat NAN Ukr 2015; 12(3): 121-132. · Zbl 1340.74004

10 Bakhvalov, NS, Panasenko, G. Homogenisation: Averaging Processes in Periodic Media: Mathematical Problems in the Mechanics of Composite Materials, Mathematics and Its Applications, Soviet Series Vol. 36. Dordrecht, The Netherlands: Kluwer Academic Publishers, 1989.

11 Jikov, VV, Kozlov, SM, Olejnik, OA. Homogenization of differential operators and integral functionals. Berlin: Springer Verlag, 1994.

12 Grigolyuk, EhI, Fil'shtinskij, LA. Periodic piecewise homogeneous elastic structures (in Russian). Moscow: Nauka, 1992.

13 Kachanov, M, Sevostianov, I. On quantitative characterization of microstructures and effective properties. Int J Solid Struct 2005; 42: 309-336. · Zbl 1101.74013

14 Kuhlmey, BT, White, TH, Renversez, G. Multipole method for microstructured optical fibers. II. Implementation and results. J Opt Soc Am B 2002; 19(10): 2331-2340.

15 Milton, GW. The theory of composites. Cambridge: Cambridge University Press, 2002. · Zbl 0993.74002

16 Sangani, AS, Acrivos, A. Slow flow past periodic arrays of cylinders with application to heat transfer. Int J Multiph Flow 1982; 8(3): 193-206. · Zbl 0487.76048

17 White, TP, Kuhlmey, BT, McPhedran, RC. Multipole method for microstructured optical fibers. I. Formulation. J Opt Soc Am B 2002; 19(10): 2322-2330.

18 Bensoussan, A, Lions, J-L, Papanicolaou, G. Asymptotic analysis for periodic structures. Amsterdam: Elsevier, 1978. · Zbl 0404.35001

19 Castro, LP, Kapanadze, D, Pesetskaya, E. A heat conduction problem of 2D unbounded composites with imperfect contact conditions. Z Angew Math Mech 2015; 95(9): 952-965. · Zbl 1326.74037

20 Kapanadze, D, Mishuris, G, Pesetskaya, E. Improved algorithm for analytical solution of the heat conduction problem in composites. Compl Var Ellipt Eq 2015; 60(1): 1-23. · Zbl 1316.30041

21 McPhedran, RC, McKenzie, DR. The conductivity of lattices of spheres. I. The simple cubic lattice. Proc R Soc Lond A Math Phys Sci 1978; 359(1696): 45-63.

22 Perrins, WT, McKenzie, DR, McPhedran, RC. Transport properties of regular arrays of cylinders. Proc R Soc Lond A Math Phys Sci 1979; 369(1737): 207-225.

23 Kushch, V. Micromechanics of composites: Multipole expansion approach. Amsterdam: Butterworth-Heinemann, 2013.

24 Benveniste, Y, Miloh, T. Imperfect soft and stiff interfaces in two-dimensional elasticity. Mech Mater 2001; 33: 309-323.

25 Kirchhoff, GR. Teorie der wärme. Leipzig: Druck und Verlag von B.G.Teubner, 1894.

26 Snarskii, AA, Zhenirovskiy, M. Effective conductivity of non-linear composites. Phys B 2002; 322: 84-91.

27 Kapanadze, D, Mishuris, G, Pesetskaya, E. Exact solution of a nonlinear heat conduction problem in a doubly periodic 2D composite material. Arch Mech 2015; 67(2): 157-178. · Zbl 1329.80005

28 Castro, LP, Kapanadze, D, Pesetskaya, E. Effective conductivity of a composite material with stiff imperfect contact conditions. Math Meth Appl Sci 2015; 38(18): 4638-4649. · Zbl 1345.31005