Bazhenov, Nikolay A.; Kalimullin, Iskander Sh.; Yamaleev, Mars M.
Degrees of categoricity and spectral dimension. (English) Zbl 1447.03008
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Summary: A Turing degree **d** is the degree of categoricity of a computable structure $\mathcal{S}$ if **d** is the least degree capable of computing isomorphisms among arbitrary computable copies of $\mathcal{S}$. A degree **d** is the strong degree of categoricity of $\mathcal{S}$ if **d** is the degree of categoricity of $\mathcal{S}$, and there are computable copies $\mathcal{A}$ and $\mathcal{B}$ of $\mathcal{S}$ such that every isomorphism from $\mathcal{A}$ onto $\mathcal{B}$ computes **d**. In this paper, we build a c.e. degree **d** and a computable rigid structure $\mathcal{M}$ such that **d** is the degree of categoricity of $\mathcal{M}$, but **d** is not the strong degree of categoricity of $\mathcal{M}$. This solves the open problem of *E. B. Fokina* et al. [Arch. Math. Logic 49, No. 1, 51–67 (2010; Zbl 1184.03026)].

For a computable structure $\mathcal{S}$, we introduce the notion of the spectral dimension of $\mathcal{S}$, which gives a quantitative characteristic of the degree of categoricity of $\mathcal{S}$. We prove that for a nonzero natural number N, there is a computable rigid structure $\mathcal{M}$ such that $0'$ is the degree of categoricity of $\mathcal{M}$, and the spectral dimension of $\mathcal{M}$ is equal to N.

MSC:

03D45 Theory of numerations, effectively presented structures
03C57 Computable structure theory, computable model theory
03C35 Categoricity and completeness of theories
03D28 Other Turing degree structures

Keywords:

categoricity spectrum; degree of categoricity; rigid structure; computable categoricity

Full Text: DOI

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