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On the stability of Thomson's vortex N-gon and a vortex tripole/quadrupole in geostrophic models of Bessel vortices and in a two-layer rotating fluid: a review. (English) Zbl 1446.76114

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Summary: In this paper the two-layer geostrophic model of the rotating fluid and the model of Bessel vortices are considered. Kirchhoff's model of vortices in a homogeneous fluid is the limiting case of both of these models. Part of the study is performed for an arbitrary Hamiltonian depending on the distances between point vortices.

The review of the stability problem of stationary rotation of regular Thomson's vortex N-gon of identical vortices is given for $N \geqslant 2$. The stability problem of the vortex tripole/quadrupole is also considered. This axisymmetric vortex structure consists of a central vortex of an arbitrary intensity and two/three identical peripheral vortices. In the model of a two-layer fluid, peripheral vortices belong to one of the layers and the central vortex can belong to either another layer or the same.

The stability of the stationary rotation is interpreted as orbital stability (the stability of a one-parameter orbit of a stationary rotation of a vortex system). The instability of the stationary rotation is instability of equilibrium of the reduced system. The quadratic part of the Hamiltonian and eigenvalues of the linearization matrix are studied.

The parameter space is divided into three parts: **A** is the domain of stability in an exact nonlinear setting, **B** is the linear stability domain, where the stability problem requires nonlinear analysis, and **C** is the instability domain.

In the stability problem of a vortex multipole, another definition of stability is used; it is the stability of an invariant three-parametric set of all trajectories of the families of stationary orbits. It is shown that in the case of non zero total intensity, the stability of the invariant set implies orbital stability.

MSC:

76E07 Rotation in hydrodynamic stability
76B47 Vortex flows for incompressible inviscid fluids
76E20 Stability and instability of geophysical and astrophysical flows
76-02 Research exposition (monographs, survey articles) pertaining to fluid mechanics

Keywords:

point vortex; Kirchhoff model; orbital stability; Hamiltonian equation

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