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On the stability and bifurcation of the non-rotating Boussinesq equation with the Kolmogorov forcing at a low Péclet number. (English) Zbl 1444.76058

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Summary: This study examines the stability and potential bifurcations of a stratified shear flow governed by the non-rotating incompressible Boussinesq equation at a low Péclet number. For the ratio of the vertical scale to the horizontal scale of a stratified flow $a \in [\sqrt{3}/4, \sqrt{3}/2)$, it is shown that there exists a threshold for the Reynold number Re above which the steady stratified shear flow driven by the Kolmogorov forcing becomes linearly unstable. As a result, the Boussinesq equation exhibits a Hopf bifurcation. To further determine the type of the Hopf bifurcation, the model is reduced to a low-order system whose numerical analyses reveal that the Hopf bifurcation is supercritical. That is, a stable periodic solution emerges, which describes an oscillating thermal convection in a highly stratified shear flow arising in the atmosphere or interior of many stellar systems with low Péclet numbers.

MSC:

76E20 Stability and instability of geophysical and astrophysical flows
76E15 Absolute and convective instability and stability in hydrodynamic stability
37L10 Normal forms, center manifold theory, bifurcation theory for infinite-dimensional dissipative dynamical systems
35B32 Bifurcations in context of PDEs

Keywords:

Boussinesq equation; stratified flow; Hopf bifurcation; Kolmogorov forcing

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