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On the viscous instabilities and transitions of two-layer model with a layered topography.

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Summary: In this article, the viscously-damped instability arising in the shear jet of west boundary layer governed by the two-layer quasi-geostrophic equation with a layered topography is analyzed. First, the nonlinear stability and the exponential stability of the shear jet is studied. More precisely, we derive an upper bound on the Reynolds number Re below which the shear jet is not only locally nonlinearly stable but also globally exponentially stable. Second, it is shown that there exists a critical value of the Reynolds number Re above which the shear jet will become linearly unstable and there exists a dynamic transition in the west boundary layer. To shed light on the type of the dynamic transition, we reduce the two-layer quasi-geostrophic equation to a system of ODEs by making use of the technique of center manifold reduction. Then, we infer from this system of ODEs that the dynamic transition is of continuous type, leading to a stable periodic oscillation of west boundary layer currents. Finally, we investigate the effect of the slope of the bottom topography on the stability and transition of the shear jet. We find that although a large slope stabilizes the shear jet, it has no impact on the transition type.

MSC:

76D99 Incompressible viscous fluids
76E20 Stability and instability of geophysical and astrophysical flows
37L15 Stability problems for infinite-dimensional dissipative dynamical systems
37L99 Infinite-dimensional dissipative dynamical systems

Keywords:

western boundary currents; viscously-damped shear instability; continuous transition; dynamic transition; Hopf bifurcation

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