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Asymptotic stability of multi-soliton solutions for nonlinear Schrödinger equations with time-dependent potential. (English) [Zbl 1443.81029](#)

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Summary: In this paper, we consider the nonlinear Schrödinger equation $i\partial_t\psi = -\frac{1}{2}\Delta\psi + V(t, x)\psi - F(|\psi|^2)\psi$ with time-dependent potential in \mathbb{R}^3 . We prove that the weakly interacting N -soliton is asymptotically stable in a Sobolev space $H^1(\mathbb{R}^3)$ under certain assumptions on the time dependent potential $V(t, x)$ and the spectral structures of the linearized Hamiltonian.

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MSC:

- 81Q05** Closed and approximate solutions to the Schrödinger, Dirac, Klein-Gordon and other equations of quantum mechanics
- 81Q10** Selfadjoint operator theory in quantum theory, including spectral analysis
- 35Q55** NLS equations (nonlinear Schrödinger equations)
- 35Q41** Time-dependent Schrödinger equations and Dirac equations
- 35C08** Soliton solutions
- 46E39** Sobolev (and similar kinds of) spaces of functions of discrete variables
- 93B18** Linearizations

Full Text: [DOI](#)

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