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**Stability of multirate explicit coupling of geomechanics with flow in a poroelastic medium.**  
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**Summary:** We consider single rate and multirate explicit schemes for the Biot system modeling coupled flow and geomechanics in a poro-elastic medium. These schemes are widely used in practice that follows a sequential procedure in which the flow and mechanics problems are fully decoupled. In such a scheme, the flow problem is solved first with time-lagging the displacement term followed by the mechanics solve. The multirate explicit coupling scheme exploits the different time scales for the mechanics and flow problems by taking multiple finer time steps for flow within one coarse mechanics time step. We provide fully discrete schemes for both the single and multirate approaches that use Backward Euler time discretization and mixed spaces for flow and conformal Galerkin for mechanics. We perform a rigorous stability analysis and derive the conditions on reservoir parameters and the number of finer flow solves to ensure stability for both schemes. Furthermore, we investigate the computational time savings for explicit coupling schemes against iterative coupling schemes.

**MSC:**

- [65M60](#) Finite element, Rayleigh-Ritz and Galerkin methods for initial value and initial-boundary value problems involving PDEs Cited in 1 Document
- [76E20](#) Stability and instability of geophysical and astrophysical flows
- [76S05](#) Flows in porous media; filtration; seepage

**Keywords:**

[poroelasticity](#); [Biot system](#); [explicit coupling](#); [multirate scheme](#); [mixed formulation](#)

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