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On reference solutions and the sensitivity of the 2D Kelvin-Helmholtz instability problem. (English) Zbl 1442.76050

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Summary: Two-dimensional Kelvin-Helmholtz instability problems are popular examples for assessing discretizations for incompressible flows at high Reynolds number. Unfortunately, the results in the literature differ considerably. This paper presents computational studies of a Kelvin-Helmholtz instability problem with high order divergence-free finite element methods. Reference results in several quantities of interest are obtained for three different Reynolds numbers up to the beginning of the final vortex pairing. A mesh-independent prediction of the final pairing is not achieved due to the sensitivity of the considered problem with respect to small perturbations. A theoretical explanation of this sensitivity to small perturbations is provided based on the theory of self-organization of 2D turbulence. Possible sources of perturbations that arise in almost any numerical simulation are discussed.

MSC:

76D17	Viscous vortex flows
76D05	Navier-Stokes equations for incompressible viscous fluids
76E20	Stability and instability of geophysical and astrophysical flows
76F20	Dynamical systems approach to turbulence
76F65	Direct numerical and large eddy simulation of turbulence
76M10	Finite element methods applied to problems in fluid mechanics

Keywords:

Kelvin-Helmholtz instability; mixing layer; incompressible Navier-Stokes equations; direct numerical simulation; reference solutions; sensitivity with respect to components of numerical methods

Software:

Netgen

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