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Stability of the solitary manifold of the perturbed sine-Gordon equation. (English)

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Summary: We study the perturbed sine-Gordon equation $\theta_{tt} - \theta_{xx} + \sin\theta = F(\varepsilon, x)$, where F is of differentiability class C^n in ε and the first k derivatives vanish at $\varepsilon = 0$, i.e., $\partial_\varepsilon^l F(0, \cdot) = 0$ for $0 \le l \le k$. We construct implicitly a virtual solitary manifold by deformation of the classical solitary manifold in n iteration steps. Our main result establishes that the initial value problem with an appropriate initial state ε^n-close to the virtual solitary manifold has a unique solution, which follows up to time $1/(\widetilde{C}\varepsilon^{\frac{k+1}{2}})$ and errors of order ε^n a trajectory on the virtual solitary manifold. The trajectory on the virtual solitary manifold is described by two parameters, which satisfy a system of ODEs. In contrast to previous works our stability result yields arbitrarily high accuracy as long as the perturbation F is sufficiently often differentiable.

MSC:

35C08 Soliton solutions
35L71 Second-order semilinear hyperbolic equations
35B40 Asymptotic behavior of solutions to PDEs

Keywords:

soliton; symplectic decomposition; modulation equation; Lyapunov function

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References:

[1] Benjamin, T. Brooke, Applications of Leray-Schauder degree theory to problems of hydrodynamic stability, Math. Proc. Camb. Philos. Soc., 79, 2, 373-392 (1976) · Zbl 0351.76054

[2] Bona, J., On the stability theory of solitary waves, Proc. R. Soc. Lond. Ser. A, 344, 1638, 363-374 (1975) · Zbl 0328.76016

[3] Brezis, Haim, Functional Analysis, Sobolev Spaces and Partial Differential Equations, Universitext (2011), Springer: Springer New York · Zbl 1220.46002

[4] Buslaev, V. S.; Perel'man, G. S., On nonlinear scattering of states which are close to a soliton, Méthodes Semi-Classiques, vol. 2. Méthodes Semi-Classiques, vol. 2, Nantes, 1991. Méthodes Semi-Classiques, vol. 2. Méthodes Semi-Classiques, vol. 2, Nantes, 1991, Astérisque, (210):6, 49-63 (1992) · Zbl 0795.35111

[5] Côte, Raphaël; Muñoz, Claudio; Pilod, Didier; Simpson, Gideon, Asymptotic stability of high-dimensional Zakharov-Kuznetsov solitons, Arch. Ration. Mech. Anal., 220, 2, 639-710 (2016) · Zbl 1334.35276

[6] Deimling, Klaus, Nonlinear Functional Analysis (1985), Springer-Verlag: Springer-Verlag Berlin · Zbl 0559.47040

[7] Frenkel, Y. I.; Kontorova, T., J. Phys. Acad. Sci. USSR, 1, 137 (1939)

[8] Fröhlich, J.; Gustafson, S.; Jonsson, B. L.G.; Sigal, I. M., Solitary wave dynamics in an external potential, Commun. Math. Phys., 250, 3, 613-642 (2004) · Zbl 1075.35075

[9] Henry, Daniel B.; Perez, J. Fernando; Wreszinski, Walter F., Stability theory for solitary-wave solutions of scalar field equations, Commun. Math. Phys., 85, 3, 351-361 (1982) · Zbl 0546.35062

[10] Hislop, P. D.; Sigal, I. M., Introduction to Spectral Theory, Applied Mathematical Sciences, vol. 113 (1996), Springer-Verlag: Springer-Verlag New York, with applications to Schrödinger operators · Zbl 0855.47002

[11] Holmer, Justin, Dynamics of KdV solitons in the presence of a slowly varying potential, Int. Math. Res. Not., 23, 5367-5397 (2011) · Zbl 1247.35132

[12] Holmer, Justin; Lin, Quanhui, Phase-driven interaction of widely separated nonlinear Schrödinger solitons, J. Hyperbolic Differ. Equ., 9, 3, 511-543 (2012) · Zbl 1256.35137

[13] Holmer, Justin; Zworski, Maciej, Slow soliton interaction with delta impurities, J. Mod. Dyn., 1, 4, 689-718 (2007) · Zbl 1137.35060

[14] Holmer, Justin; Zworski, Maciej, Soliton interaction with slowly varying potentials, Int. Math. Res. Not., 10, Article rnn026 pp. (2008) · Zbl 1147.35084

[15] Imaykin, Valery; Komech, Alexander; Vainberg, Boris, Scattering of solitons for coupled wave-particle equations, J. Math.

Anal. Appl., 389, 2, 713-740 (2012) · Zbl 1235.35068

[16] Inoue, Masahiro; Chung, S. G., Bion dissociation in sine-Gordon system, J. Phys. Soc. Jpn., 46, 5, 1594-1601 (1979)

[17] Kivshar, Yuri S.; Malomed, Boris A., Dynamics of solitons in nearly integrable systems, Rev. Mod. Phys., 61, 763-915 (Oct 1989)

[18] Komech, Alexander; Spohn, Herbert; Kunze, Markus, Long-time asymptotics for a classical particle interacting with a scalar wave field, Commun. Partial Differ. Equ., 22, 1-2, 307-335 (1997) · Zbl 0878.35094

[19] Kopylova, Elena, Asymptotic Stability of Solitons for Nonlinear Hyperbolic Equations (2015), Universität Wien, Habilitationsschrift · Zbl 1327.35266

[20] Kowalczyk, Michal; Martel, Yvan; Muñoz, Claudio, Kink dynamics in the \(\phi^4\) model: asymptotic stability for odd perturbations in the energy space, J. Am. Math. Soc., 30, 3, 769-798 (2017) · Zbl 1387.35419

[21] Lars, B.; Jonsson, G.; Fröhlich, Jürg; Gustafson, Stephen; Sigal, Israel Michael, Long time motion of NLS solitary waves in a confining potential, Ann. Henri Poincaré, 7, 4, 621-660 (2006) · Zbl 1100.81019

[22] Martin, Robert H., Nonlinear Operators and Differential Equations in Banach Spaces, Pure and Applied Mathematics (1976), Wiley-Interscience [John Wiley \& Sons]: Wiley-Interscience [John Wiley \& Sons] New York-London-Sydney · Zbl 0333.47023

[23] Mashkin, Timur, Stability of the Solitary Manifold of the Sine-Gordon Equation (2016), Universität zu Köln

[24] Mashkin, Timur, Invariant virtual solitary manifold of the perturbed sine-Gordon equation (2018)

[25] Mikeska, H. J., Solitons in a one-dimensional magnet with an easy plane, J. Phys. C, Solid State Phys., 11, 1, L29 (1978)

[26] Mizumachi, Tetsu; Pelinovsky, Dmitry, Bäcklund transformation and \(L^2\)-stability of NLS solitons, Int. Math. Res. Not., 9, 2034-2067 (2012) · Zbl 1239.35148

[27] Skyrme, T. H.R., Particle states of a quantized meson field, Proc. R. Soc. Lond., Ser. A, Math. Phys. Eng. Sci., 262, 1309, 237-245 (1961) · Zbl 0099.43605

[28] Soffer, A.; Weinstein, M. I., Multichannel nonlinear scattering for nonintegrable equations, Commun. Math. Phys., 133, 1, 119-146 (1990) · Zbl 0721.35082

[29] Stuart, David M. A., Perturbation theory for kinks, Commun. Math. Phys., 149, 3, 433-462 (1992) · Zbl 0756.35084

[30] Stuart, David M., Solitons on pseudo-Riemannian manifolds. I. The sine-Gordon equation, Commun. Partial Differ. Equ., 23, 9-10, 1815-1837 (1998) · Zbl 0935.35143

[31] David M.A. Stuart, Sine Gordon notes, unpublished notes, 2012.

[32] Weinstein, Michael I., Lyapunov stability of ground states of nonlinear dispersive evolution equations, Commun. Pure Appl. Math., 39, 1, 51-67 (1986) · Zbl 0594.35005

[33] Zhang, L.; Huang, L.; Qiu, X. M., Josephson junction dynamics in the presence of microresistors and an ac drive, J. Phys. Condens. Matter, 7, 2, 353 (1995)