Han, Zhilin; **Stoter, Stein K. F.**; **Wu, Chien-Ting**; **Cheng, Changzheng**; **Mantzaflaris, Angelos**; **Mogilevskaya, Sofia G.**; **Schillinger, Dominik**

Consistent discretization of higher-order interface models for thin layers and elastic material surfaces, enabled by isogeometric cut-cell methods. (English) Zbl 1441.74031

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Summary: Many interface formulations, e.g. based on asymptotic thin interphase models or material surface theories, involve higher-order differential operators and discontinuous solution fields. In this article, we are taking first steps towards a variationally consistent discretization framework that naturally accommodates these two challenges by synergistically combining recent developments in isogeometric analysis and cut-cell finite element methods. Its basis is the mixed variational formulation of the elastic interface problem that provides access to jumps in displacements and stresses for incorporating general interface conditions. Upon discretization with smooth splines, derivatives of arbitrary order can be consistently evaluated, while cut-cell meshes enable discontinuous solutions at potentially complex interfaces. We demonstrate via numerical tests for three specific nontrivial interfaces (two regimes of the Benveniste-Miloh classification of thin layers and the Gurtin-Murdoch material surface model) that our framework is geometrically flexible and provides optimal higher-order accuracy in the bulk and at the interface.

MSC:

74A50 Structured surfaces and interfaces, coexistent phases
65D07 Numerical computation using splines
74S05 Finite element methods applied to problems in solid mechanics
65N30 Finite element, Rayleigh-Ritz and Galerkin methods for boundary value problems involving PDEs

Keywords:

asymptotic models of thin interphases; theories of material surfaces; variational interface formulations; isogeometric analysis; cut-cell finite element methods

Software:

CutFEM

Full Text: DOI

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