Borwein, Jonathan M.; **Sims, Brailey**; **Tam, Matthew K.**

Norm convergence of realistic projection and reflection methods. (English) Zbl 1440.47050

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In Hilbert spaces, fixed point algorithms that exhibit a Fejér-monotone behavior converge usually only weakly and not strongly. Specific examples of such asymptotic patterns can be found in [*A. Genel* and *J. Lindenstrauss*, Isr. J. Math. 22, 81–86 (1975; Zbl 0314.47031); *O. Güler*, SIAM J. Control Optim. 29, No. 2, 403–419 (1991; Zbl 0737.90047); *H. S. Hundal*, Nonlinear Anal., Theory Methods Appl., Ser. A, Theory Methods 57, No. 1, 35–61 (2004; Zbl 1070.46013); *H. H. Bauschke* et al., Nonlinear Anal., Theory Methods Appl., Ser. A, Theory Methods 56, No. 5, 715–738 (2004; Zbl 1059.47060); *P. L. Combettes* and *V. R. Wajs*, Multiscale Model. Simul. 4, No. 4, 1168–1200 (2005; Zbl 1179.94031); *H. H. Bauschke* et al., Proc. Am. Math. Soc. 133, No. 6, 1829–1835 (2005; Zbl 1071.65082)]. While Fejér-monotone algorithms can be turned into strongly convergent ones via a Haugazeau-like transformation [*H. H. Bauschke* and *P. L. Combettes*, Math. Oper. Res. 26, No. 2, 248–264 (2001; Zbl 1082.65058)], it remains an open problem to derive new sufficient conditions under which the unaltered iteration converges strongly. The authors address this question by focusing on projections methods for the basic 2-set convex feasibility problem in a Hilbert space and, in particular, on the Douglas-Rachford method, for which little is known in terms of strong convergence conditions. Using recession analysis, new strong convergence conditions are obtained for the Douglas-Rachford and alternating projection algorithms. The strong convergence of a relaxed version of the Douglas-Rachford algorithm is also investigated. The paper concludes with an analysis of the possible failure of linear convergence of the Douglas-Rachford and alternating projection algorithms.

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MSC:

47J25 Iterative procedures involving nonlinear operators
47H09 Contraction-type mappings, nonexpansive mappings, A-proper mappings, etc.
90C25 Convex programming
90C48 Programming in abstract spaces

Keywords:

projection methods; reflection methods; alternating projection method; Douglas-Rachford method; strong convergence; norm convergence; Hilbert lattice

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