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**Weak ideal topology in the topos of right acts over a monoid.** (English) Zbl 1439.18004  
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**Summary:** Let  $S$  be a monoid. In this manuscript, our purpose is to study the notion of weak ideal topology  $j^I$  on the topos  $\mathbf{Act}\text{-}S$  of all (right) representations of  $S$ , where  $I$  is a left ideal of  $S$ . After a brief analysis of the weak ideal topology, we give a necessary and sufficient condition for a  $j^I$ -separated  $S$ -act to become a  $j^I$ -sheaf in which the ideal  $I$  is central. Moreover, we establish another form of the double negation topology on  $\mathbf{Act}\text{-}S$  which we call the torsion topology. Then, we retrieve the torsion topology on  $\mathbf{Act}\text{-}S$  by means of the internal existential quantifier  $\exists_l : \mathbf{Rld}(S)^I \rightarrow \mathbf{Rld}(S)$ , in which  $\mathbf{Rld}(S)$  is the Heyting algebra of all right ideals of  $S$ . Furthermore we give an explicit description of the associated sheaf functor for the ideal topology  $j^I$  where  $I$  is a central band of  $S$ ; e.g. the ideal  $\mathbb{N}$  of natural numbers of the monoid  $(\mathbb{N}^\infty, \min)$  of extended natural numbers. Finally, for certain ideals  $I$  we show that the topos of all  $j^I$ -sheaves is a De Morgan topos provided that the monoid  $S$  satisfies in the right Ore condition.

**MSC:**

- 18B25 Topoi
- 06A15 Galois correspondences, closure operators (in relation to ordered sets)
- 20M12 Ideal theory for semigroups
- 20M30 Representation of semigroups; actions of semigroups on sets
- 20M50 Connections of semigroups with homological algebra and category theory

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**Keywords:**

associated sheaf functor; double negation topology; separated  $S$ -act; (weak) ideal topology

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