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Completely distributive completions of posets. (English) Zbl 1438.06011

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In a poset P , let \mathcal{F} stand for a collection of upsets containing all principal upsets and \mathcal{I} , for a collection of downsets containing all principal downsets. The author deals with $(\mathcal{F}, \mathcal{I})$ -completions in the sense of *M. Gehrke* et al. [*Order* 30, No. 1, 39–64 (2013; [Zbl 1317.06002](#))]. The results presented in the paper under review are organized in several groups.

(1) If the pair $(\mathcal{F}, \mathcal{I})$ has a certain separation property (P) (for every $F \in \mathcal{F}$ and every $I \in \mathcal{I}$, if $F \cap I = \emptyset$, then there exists $\mathcal{H} \in \mathcal{F}$ with $F^c \in \mathcal{I}$ such that $F \subseteq \mathcal{H}$ and $\mathcal{H} \cap I = \emptyset$), then the $(\mathcal{F}, \mathcal{I})$ -completion of P is a completely distributive algebraic lattice (c.d.a.l., for short).

(2) It is shown how, for \mathcal{F} a distributive lattice, choose \mathcal{I} to force the $(\mathcal{F}, \mathcal{I})$ -completion of P to be a c.d.a.l. In this way, the canonical extension of a distributive meet semilattice is demonstrated to be such a lattice.

(3) Studied are also extensions of additional n -ary operations on P to their corresponding $(\mathcal{F}, \mathcal{I})$ -completions.

(4) The previous results are used to obtain appropriate $(\mathcal{F}, \mathcal{I})$ -completions for Tarski algebras, for Hilbert algebras, and for algebras that are associated with filter distributive finitary congruential logics. For the latter case, such extensions, known as the \mathcal{S} -canonical extensions, have already been defined and investigated by *M. Gehrke* et al. [*Ann. Pure Appl. Logic* 161, No. 12, 1502–1519 (2010; [Zbl 1238.03051](#))]. The approach developed in the paper under review, though also involving abstract algebraic logic, seems to be more straightforward.

Reviewer: [Jānis Cīrulis \(Riga\)](#)

MSC:

- 06B23 Complete lattices, completions
- 06A06 Partial orders, general
- 06A15 Galois correspondences, closure operators (in relation to ordered sets)
- 06D10 Complete distributivity
- 03G25 Other algebras related to logic

Keywords:

poset; completion; extension of maps; complete distributivity

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