

Cattaneo, Gianpiero

Algebraic methods for rough approximation spaces by lattice interior-closure operations.
(English) [Zbl 1437.03150](#)

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Summary: This chapter deals with the abstract approach to rough sets theory through the equational notion of closure operator in the context of lattice theory, with the associated notion of internal operator as not-closure-not. The involved lattice structures are not necessarily distributive to allow the development of rough theories in the so-called logical-algebraic context of Quantum Mechanics, based on non-distributive lattices of orthomodular type. The chapter is organized into four parts.

In Part I the more general lattice notion of closure operator is introduced, and the induced notion of interior operator is discussed. It is shown that this lattice approach of the inner-closure pairs is categorically equivalent to the non-equational abstract notion of approximation space based on the lower-upper approximation of each lattice element.

Part II deals with three variations of closure operators called respectively, from the more general to the stronger one, as Tarski, Kuratowski and Halmos closures. A characterization of this last is given in terms of Brouwer Zadeh lattice structure.

Part III provides an interpretation of the pairs of internal-closure operators in terms of pairs of necessity-possibility operators in the context of suitable modal logics. A Kripke-like semantic of such kinds of logics is also provided based on a set of possible worlds. The usual approach to the concrete theory of rough sets through Pawlak information systems is investigated in this context.

For the entire collection see [\[Zbl 1411.03006\]](#).

MSC:

- 03E72** Theory of fuzzy sets, etc.
- 03G10** Logical aspects of lattices and related structures
- 03G25** Other algebras related to logic
- 06A15** Galois correspondences, closure operators (in relation to ordered sets)
- 06C15** Complemented lattices, orthocomplemented lattices and posets
- 06D30** De Morgan algebras, Łukasiewicz algebras (lattice-theoretic aspects)
- 68T37** Reasoning under uncertainty in the context of artificial intelligence

Full Text: [DOI](#)

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