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The Church-Rosser theorem and quantitative analysis of witnesses. (English) Zbl 1436.03104
Inf. Comput. 263, 52-56 (2018).

Summary: We show that an upper bound function for the Church-Rosser theorem of type-free λ -calculus with β -reduction must be in the fourth level of the Grzegorzcyk hierarchy, i.e., the smallest Grzegorzcyk class properly extending the class of elementary functions. At this level we also find common reducts for the confluence property. The proof method here can be applied not only to type-free λ -calculus with $\beta\eta$ -reduction but also to typed λ -calculi such as Pure Type Systems.

MSC:

03B40 Combinatory logic and lambda calculus

Cited in 1 Document

Keywords:

lambda-calculus; Church-Rosser theorem; upper bounds on reduction length; parallel reduction; reduction strategies; Takahashi's translation; Gross-Knuth reduction strategy; Grzegorzcyk hierarchy

Software:

Automath

Full Text: [DOI](#)

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