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A new Galois structure in the category of internal preorders. (English) Zbl 1435.18009
Theory Appl. Categ. 35, 326-349 (2020).

Summary: Let $\text{PreOrd}(\mathbb{C})$ be the category of internal preorders in an exact category \mathbb{C} . We show that the pair $(\text{Eq}(\mathbb{C}), \text{ParOrd}(\mathbb{C}))$ is a pretorsion theory in $\text{PreOrd}(\mathbb{C})$, where $\text{Eq}(\mathbb{C})$ and $\text{ParOrd}(\mathbb{C})$ are the full subcategories of internal equivalence relations and of internal partial orders in \mathbb{C} , respectively. We observe that $\text{ParOrd}(\mathbb{C})$ is a reflective subcategory of $\text{PreOrd}(\mathbb{C})$ such that each component of the unit of the adjunction is a pullback-stable regular epimorphism. The reflector $F: \text{PreOrd}(\mathbb{C}) \rightarrow \text{ParOrd}(\mathbb{C})$ turns out to have stable units in the sense of Cassidy, Hébert and Kelly, thus inducing an admissible categorical Galois structure. In particular, when \mathbb{C} is the category Set of sets, we show that this reflection induces a monotone-light factorization system (in the sense of [A. Carboni et al., Appl. Categ. Struct. 5, No. 1, 1–58 (1997; Zbl 0866.18003)]) in $\text{PreOrd}(\text{Set})$. A topological interpretation of our results in the category of Alexandroff-discrete spaces is also given, via the well-known isomorphism between this latter category and $\text{PreOrd}(\text{Set})$.

MSC:

- 18E50** Categorical Galois theory
- 18A32** Factorization systems, substructures, quotient structures, congruences, amalgams
- 18B35** Preorders, orders, domains and lattices (viewed as categories)
- 18E40** Torsion theories, radicals
- 06A15** Galois correspondences, closure operators (in relation to ordered sets)

Cited in **2** Documents

Keywords:

internal preorders; partial orders; Galois theory; monotone-light factorization system; Alexandroff-discrete spaces

Full Text: [arXiv Link](#)

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