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Point vortices for inviscid generalized surface quasi-geostrophic models. (English)

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Summary: We give a rigorous proof of the validity of the point vortex description for a class of inviscid generalized surface quasi-geostrophic models on the whole plane.

MSC:

76B47 Vortex flows for incompressible inviscid fluids
76M23 Vortex methods applied to problems in fluid mechanics
76E20 Stability and instability of geophysical and astrophysical flows
86A05 Hydrology, hydrography, oceanography

Keywords:

inviscid generalized surface quasi-geostrophic; weak solutions; point vortex motion; vortex approximation; localization; stability

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