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Solution properties of a 3D stochastic Euler fluid equation. (English) Zbl 1433.60051
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Summary: We prove local well-posedness in regular spaces and a Beale-Kato-Majda blow-up criterion for a recently derived stochastic model of the 3D Euler fluid equation for incompressible flow. This model describes incompressible fluid motions whose Lagrangian particle paths follow a stochastic process with cylindrical noise and also satisfy Newton's second law in every Lagrangian domain.

MSC:

60H15 Stochastic partial differential equations (aspects of stochastic analysis) Cited in 22 Documents
35Q35 PDEs in connection with fluid mechanics
60H30 Applications of stochastic analysis (to PDEs, etc.)

Keywords:

analytical properties; stochastic fluid equations; Lie derivative estimates

Software:

jLab

Full Text: [DOI](#) [arXiv](#)

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