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Eilenberg-Mac Lane spaces for topological groups. (English) Zbl 1432.55020

Summary: In this paper, we establish a topological version of the notion of an Eilenberg-Mac Lane space. If $X$ is a pointed topological space, $\pi_1(X)$ has a natural topology coming from the compact-open topology on the space of maps $S^1 \to X$. In general, the construction does not produce a topological group because it is possible to create examples where the group multiplication $\pi_1(X) \times \pi_1(X) \to \pi_1(X)$ is discontinuous. This discontinuity has been noticed by others, for example Fabel. However, if we work in the category of compactly generated, weakly Hausdorff spaces, we may retopologise both the space of maps $S^1 \to X$ and the product $\pi_1(X) \times \pi_1(X)$ with compactly generated topologies to see that $\pi_1(X)$ is a group object in this category. Such group objects are known as $k$-groups. Next we construct the Eilenberg-Mac Lane space $K(G,1)$ for any totally path-disconnected $k$-group $G$. The main point of this paper is to show that, for such a $G$, $\pi_1(K(G,1))$ is isomorphic to $G$ in the category of $k$-groups. All totally disconnected locally compact groups are $k$-groups and so our results apply in particular to profinite groups, answering a question of Sauer’s. We also show that analogues of the Mayer-Vietoris sequence and Seifert-van Kampen theorem hold in this context. The theory requires a careful analysis using model structures and other homotopical structures on cartesian closed categories as we shall see that no theory can be comfortably developed in the classical world.

MSC:
55P20 Eilenberg-Mac Lane spaces
18N40 Homotopical algebra, Quillen model categories, derivators
18N50 Simplicial sets, simplicial objects

Keywords:
Eilenberg-Mac Lane space; $k$-group; homotopical algebra

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