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Classical mirror symmetry. (English) [Zbl 1431.14034](#)

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Mirror symmetry – the equivalence of superconformal field theories associated to mirror dual pairs of Calabi-Yau threefolds – remains one of the most productive, and profound, points of contact between theoretical physics and mathematics. Indeed, a wide array of disciplines within algebraic and symplectic geometry have been heavily influenced by insights derived from mirror symmetry. Famous examples of these include Gromov-Witten theory, the study of derived categories of sheaves and their stability conditions, special Lagrangian torus fibrations, Lagrangian Floer theory, and the theory of cluster algebras.

‘Classical Mirror Symmetry’ provides a concise account of the origins and early results in this subject, providing a synthesis of the string theory background of the subject and the surrounding mathematics. Given that familiarity with supersymmetric quantum field theories is assumed, graduate students – or other researchers – in theoretical physics with an interest in the mathematical aspects of mirror symmetry form a natural audience for this text. However, for purely mathematical students of mirror symmetry, this book could be studied alongside the seminal Clay monograph, [*K. Hori et al.*, *Mirror symmetry*. Providence, RI: American Mathematical Society (AMS) (2003; [Zbl 1044.14018](#))].

As well as summarising the physical and mathematical backgrounds to mirror symmetry, the book aims to provide an novel remodelling of the mathematical formulation of mirror symmetry for the quintic threefold famously established by *A. B. Givental* [*Int. Math. Res. Not.* 1996, No. 13, 613–663 (1996; [Zbl 0881.55006](#))] and *B. H. Lian et al.* [*Asian J. Math.* 1, No. 4, 729–763 (1997; [Zbl 0953.14026](#))]. In particular, the first four chapters of the book culminate in a description of Yukawa couplings for the the A and B-model topological sigma models, while the fifth pursues the authors perspective on the mathematical justification of closed string mirror symmetry.

In Chapter 1, the author provides a brief survey the physics of mirror symmetry, as described by *M. B. Green et al.* [*Superstring theory. Volume 1: Introduction. Volume 2: Loop amplitudes, anomalies and phenomenology*. Reprint of the 1987 hardback edition. Cambridge: Cambridge University Press (2012; [Zbl 1245.53003](#))], and outlines the most famous mathematical consequences of this symmetry: the exchange of Hodge numbers and the predicted count of rational curves in a quintic threefold made by *P. Candelas et al.* [*AMS/IP Stud. Adv. Math.* 9, 31–95 (1998; [Zbl 0904.32019](#))].

Chapter 2, and the first part of Chapter 4, are devoted to key aspects of the mathematical background to mirror symmetry. In particular Chapter 2 is devoted to a treatment of Kähler manifolds, and to vector bundles on these manifolds, while Chapter 4 opens with an survey of the theory of toric geometry and the work of *V. V. Batyrev* [*J. Algebr. Geom.* 3, No. 3, 493–535 (1994; [Zbl 0829.14023](#))] on the connection between mirror symmetry and the polar duality of reflexive polytopes.

Chapter 3 forms the bridge between the mathematics and physics described to this point, by introducing the details of the A-model and B-model topological sigma models as introduced by *E. Witten* [*AMS/IP Stud. Adv. Math.* 9, 121–160 (1998; [Zbl 0904.58009](#))]. This chapter surveys both the A and B twists of the $N = (2, 2)$ supersymmetric sigma model and gives a description of the correlation functions for both the A and B models. Details of the B-model computation in the case of the quintic threefold are given in the latter part of Chapter 4.

As mentioned above, Chapter 5 provides a detailed technical treatment of the mathematical justification of closed string mirror symmetry, adapting the methods of Givental and Lian-Liu-Yau.

Reviewer: [Thomas Prince \(Oxford\)](#)

MSC:

- [14J33](#) Mirror symmetry (algebro-geometric aspects)
- [81T30](#) String and superstring theories; other extended objects (e.g., branes) in quantum field theory
- [81T40](#) Two-dimensional field theories, conformal field theories, etc. in quantum mechanics
- [14J32](#) Calabi-Yau manifolds (algebro-geometric aspects)

Keywords:

mirror symmetry; Calabi-Yau manifolds; sigma models; Gromov-Witten theory

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