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**Experimental study of the nonlinear saturation of the elliptical instability: inertial wave turbulence versus geostrophic turbulence.** (English) [Zbl 1430.76192](#)  
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**Summary:** In this paper, we present an experimental investigation of the turbulent saturation of the flow driven by the parametric resonance of inertial waves in a rotating fluid. In our set-up, a half-metre wide ellipsoid filled with water is brought to solid-body rotation, and then undergoes sustained harmonic modulation of its rotation rate. This triggers the exponential growth of a pair of inertial waves via a mechanism called the libration-driven elliptical instability. Once the saturation of this instability is reached, we observe a turbulent state for which energy is injected into the resonant inertial waves only. Depending on the amplitude of the rotation rate modulation, two different saturation states are observed. At large forcing amplitudes, the saturation flow mainly consists of a steady, geostrophic anticyclone. Its amplitude vanishes as the forcing amplitude is decreased while remaining above the threshold of the elliptical instability. Below this secondary transition, the saturation flow is a superposition of inertial waves which are in weakly nonlinear resonant interaction, a state that could asymptotically lead to inertial wave turbulence. In addition to being a first experimental observation of a wave-dominated saturation in unstable rotating flows, the present study is also an experimental confirmation of the model of *T. Le Reun* et al. [“Inertial wave turbulence driven by elliptical instability”, *Phys. Rev. Lett.* 119, No. 3, Article ID 034502, 6 p. (2017; [doi:10.1103/PhysRevLett.119.034502](#))] who introduced the possibility of these two turbulent regimes. The transition between these two regimes and their relevance to geophysical applications are finally discussed.

**MSC:**

**76E20** Stability and instability of geophysical and astrophysical flows  
**76F25** Turbulent transport, mixing  
**76U05** General theory of rotating fluids  
**76E07** Rotation in hydrodynamic stability

Cited in **10** Documents

**Keywords:**

waves in rotating fluids; parametric instability

**Software:**

SciPy

**Full Text:** [DOI](#) [arXiv](#)

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