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Zariski-like topologies for lattices with applications to modules over associative rings. (English) Zbl 1430.06005

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If $\mathbb{X} = \langle X; \tau \rangle$ is a T_0 topological space, and K is the lattice of closed subsets of X, then the dual lattice $L = K^\partial$ is a complete lattice which has a subset $\tilde{X} = \{\{p\} \mid p \in X\} \subseteq L$ which, when equipped with the Zariski topology, is a space naturally homeomorphic to $\mathbb{X}$. That is, the closed subsets of $\tilde{X}$ in the Zariski topology are the sets $V(a) = \{\{p\} \in \tilde{X} \mid a \leq_L \{p\}\} = \{\{p\} \in \tilde{X} \mid p \in a\}$, and the map $p \mapsto \{p\}$ is a homeomorphism from $\mathbb{X}$ to $\tilde{\mathbb{X}}$.

This paper starts with an arbitrary complete lattice L and a proper subset $X \subsetneq L$ and creates a space on X with the sets $V(a) = \{p \in X \mid a \leq_L p\}$. L is called X-top if the collection of $V(a)$'s is closed under union, in which case they form the closed sets of a topology on X. The paper studies the topological properties of the resulting space (separation axioms, connectedness, and compactness), and provides sufficient conditions for the space to be spectral.

Reviewer: Keith Kearnes (Boulder)

MSC:

06B30 Topological lattices
06A15 Galois correspondences, closure operators (in relation to ordered sets)
16D10 General module theory in associative algebras
54B99 Basic constructions in general topology

Keywords:

topological lattices; prime modules; first submodules; strongly hollow submodules; Zariski topology; dual Zariski topology.

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