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**On the time to ruin for a dependent delayed capital injection risk model.** (English)

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Summary: In this paper, we propose a generalisation to the Cramér-Lundberg risk model, by allowing for a delayed receipt of the required capital injections whenever the surplus of an insurance firm is negative. Delayed capital injections often appear in practice due to the time taken for administrative and processing purposes of the funds from a third party or the shareholders of an insurance firm. The delay time of the capital injection depends on a critical value of the deficit in the following way: if the deficit of the firm is less than the fixed critical value, then it can be covered by available funds and therefore the required capital injection is received instantaneously. On the other hand, if the deficit of the firm exceeds the fixed critical value, then the funds are provided by an alternative source and the required capital injection is received after some time delay. In this modified model, we derive a Fredholm integral equation of the second kind for the ultimate ruin probability and obtain an explicit expression in terms of ruin quantities for the Cramér-Lundberg risk model. In addition, we show that other risk quantities, namely the expected discounted accumulated capital injections and the expected discounted overall time in red, up to the time of ruin, satisfy a similar integral equation, which can also be solved explicitly. Finally, we extend the capital injection delayed risk model, such that the delay of the capital injections depends explicitly on the amount of the deficit. In this generalised risk model, we derive another Fredholm integral equation for the ultimate ruin probability, which is solved in terms of a Neumann series.

**MSC:**

91B05 Risk models (general)

45B05 Fredholm integral equations

60G40 Stopping times; optimal stopping problems; gambling theory

91G05 Actuarial mathematics

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ruin probability; deficit dependent delayed capital injections; Fredholm integral equation; Neumann series solution

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