Sobota, Damian

Families of sets related to Rosenthal's lemma. (English) Zbl 1429.03155

Arch. Math. Logic 58, No. 1-2, 53-69 (2019).

A matrix of non-negative reals $\langle m_n^k : n,k \in \omega\rangle$ is said to be a Rosenthal matrix, if $\sum_{n\in\omega} m_n^k \leq 1$ for every $k \in \omega$. A family $\mathcal{F} \subseteq [\omega]^\omega$ is called Rosenthal if for every Rosenthal matrix $\langle m_n^k : n,k \in \omega\rangle$ and every $\varepsilon > 0$ there exists $A \in \mathcal{F}$ such that for every $k \in \omega$, $\sum_{n\in A\setminus\{k\}} m_n^k < \varepsilon$. This notion was obtained by analysing the proof of Rosenthal's lemma setting $m_n^k = \mu_k(a_n)$ for an antichain $\langle a_n : n \in \omega\rangle$ and a bounded sequence of finitely additive non-negative measures $\langle \mu_k : k \in \omega\rangle$ in an arbitrary Boolean algebra. In this setting, Rosenthal's lemma states that $[\omega]^\omega$ is a Rosenthal family. The paper under the review solves the question whether a given family $\mathcal{F}$ is a Rosenthal family. The author proves the following results: The cardinality of a Rosenthal family cannot be less than the covering of the category $\mathrm{cov}(\mathcal{M})$ and every base of a selective ultrafilter is a Rosenthal family. Under Martin's axiom for σ-centered partially ordered sets there exists a non-selective ultrafilter which is a Rosenthal family (in fact it is a P-point that is not a Q-point). The iterated Sacks forcing of length ω_2 provides a model of ZFC in which there exists a Rosenthal family of cardinality $< \mathfrak{c}$.

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MSC:

03E17	Cardinal characteristics of the continuum
28A33	Spaces of measures, convergence of measures
28A60	Measures on Boolean rings, measure algebras
03E35	Consistency and independence results
03E75	Applications of set theory
05C55	Generalized Ramsey theory

Keywords:

Rosenthal's lemma; ultrafilters; selective ultrafilters; P-points; Q-points

Full Text: DOI

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