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Zipf-Mandelbrot law, properties and its generalizations. (English) Zbl 1428.26028


Summary: Despite a wide spread applications of Zipf-Mandelbrot law, there is quite small amount of results concerning analytical properties on distribution law. On the first stage, we examine some monotonicity properties of the law, we derive the whole variety of its lower and upper estimations. We then further refine our results using some well-known inequalities such as Hölder and Lyapunov inequality.

On the second stage we consider the case when total mass of Zipf-Mandelbrot law is spread all over positive integer, and then we come to Hurwitz $\zeta$-function. As we show, it is very natural first to examine properties of Hurwitz $\zeta$-function to derive properties of Zipf-Mandelbrot law. Using some well-known inequalities such as Chebyshev’s and Lyapunov’s inequality we are able to deduce a whole variety of theoretical characterizations that include, among others, log-convexity, log-subadditivity, exponential convexity.

On the third stage, we generalize Zipf-Mandelbrot law using maximization of Shannon entropy, as we get hybrid Zipf-Mandelbrot law. It is interesting that examination of its densities provides some new insights of Lerch’s transcendent.

For the entire collection see [Zbl 1426.26002].

MSC:

26D10 Inequalities involving derivatives and differential and integral operators
94A17 Measures of information, entropy