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Iterative TV-regularization of grey-scale images. (English. Russian original) [Zbl 1427.94013](#)
J. Math. Sci., New York 242, No. 2, 323-336 (2019); translation from *Probl. Mat. Anal.* 99, 127-137 (2019).

Summary: The TV-regularization method due to *L. I. Rudin* et al. [*Physica D* 60, No. 1–4, 259–268 (1992; [Zbl 0780.49028](#))] is widely used in mathematical image analysis. We consider a nonstationary and iterative variant of this approach and provide a mathematical theory that extends the results of *E. Radmoser* et al. to the BV setting [“Scale-space properties of nonstationary iterative regularization methods”, *J. Vis. Commun. Image Represent.* 11, No. 2, 96–114 (2000; [doi:10.1006/jvci.1999.0437](#))]. While existence and uniqueness, a maximum-minimum principle, and preservation of the average grey value are not hard to prove, we also establish the convergence to a constant steady state and consider a large family of Lyapunov functionals. These properties allow us to interpret the iterated TV-regularization as a time-discrete scale-space representation of the original image.

MSC:

[94A08](#) Image processing (compression, reconstruction, etc.) in information and communication theory
[68U10](#) Computing methodologies for image processing
[49N60](#) Regularity of solutions in optimal control

Full Text: [DOI](#)

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