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Explicit and exact solutions concerning the antarctic circumpolar current with variable density in spherical coordinates. (English) Zbl 1427.76076

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Summary: We use spherical coordinates to devise a new exact solution to the governing equations of geophysical fluid dynamics for an inviscid and incompressible fluid with a general density distribution and subjected to forcing terms. The latter are of paramount importance for the modeling of realistic flows, that is, flows that are observed in some averaged sense in the ocean. Owing to the employment of spherical coordinates we do not need to resort to approximations (e.g., of f- and β-plane type) that simplify the geometry in the governing equations. Our explicit solution represents a steady purely azimuthal stratified flow with a free surface that – thanks to the inclusion of forcing terms and the consideration of the Earth's geometry via spherical coordinates – makes it suitable for describing the Antarctic Circumpolar Current and enables an in-depth analysis of the structure of this flow. In line with the latter aspect, we employ functional analytical techniques to prove that the free surface distortion is defined in a unique and implicit way by means of the pressure applied at the free surface. We conclude our discussion by setting out relations between the monotonicity of the surface pressure and the monotonicity of the surface distortion that concur with the physical expectations.

MSC:

76E20 Stability and instability of geophysical and astrophysical flows
76B07 Free-surface potential flows for incompressible inviscid fluids
86A05 Hydrology, hydrography, oceanography

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