Jiménez-Pastor, Antonio; **Pillwein, Veronika**

A computable extension for D-finite functions: DD-finite functions. (English) Zbl 1427.13034

J. Symb. Comput. 94, 90-104 (2019).

Let K be a field of characteristic zero, $K[[x]]$ the ring of formal power series over K and ∂ the standard derivation in $K[[x]]$. In addition, let R be a non-trivial differential subring of $K[[x]]$ and $R[\partial]$ the ring of linear differential operators over R. A power series $f \in K[[x]]$ is called differentially definable over R if there is a non-zero operator $A \in R[\partial]$ such that $A \cdot f = 0$. Furthermore, if R is the polynomial ring $K[x]$, then f is called D-finite. Finally, if R is the set of D-finite functions, then f is called DD-finite.

D-finite functions satisfy several closure properties. In this paper, the authors derive the analogous closure properties for DD-finite functions. In addition, it is proved that the function $\tan(x)$ is DD-finite and illustrated the execution of closure properties for this function. At the end, they address the issue of initial values $(f(0), f'(0), f''(0), \ldots)$ to define the solution within $K[[x]]$ of the given linear differential equation uniquely.

Reviewer: Amir Hashemi (Isfahan)

MSC:

13N15 Derivations and commutative rings
68W30 Symbolic computation and algebraic computation
34A25 Analytical theory of ordinary differential equations: series, transformations, transforms, operational calculus, etc.
13F25 Formal power series rings

Keywords:

holonomic functions; closure properties; formal power series

Software:

gfun; ore$_a$*lgebra*; DLMF; GeneratingFunctions; SageMath

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