
Some results of S. N. Bernstein about polynomials of least deviation from zero in some weighted spaces $L_p$ are generalized to entire functions of exponential type. Suppose a function $\rho_m$ belongs to the Cartwright class, is of type $m$, is positive on the real axis, and satisfies

$$
\rho_m(z) = \prod_{n=1}^{\infty} \left(1 - \frac{z}{z_n}\right) \left(1 - \frac{\overline{z}}{\overline{z}_n}\right),
$$

where $\text{Im} z_n > 0$ and $\sum_n \frac{\text{Im} z_n}{|z_n|^2} < \infty$. Let $\sigma \geq m$,

$$
h_m(z) = \prod_{n} \left(1 - \frac{z}{z_n}\right), \quad g_m(z) = e^{i\sigma z} h_m(z), \quad G(z) = e^{-i\sigma z} g_m^2(z).
$$

Then entire functions $f_\sigma$ and $F_\sigma$ are defined by the formulas

$$
f_\sigma(z) = \frac{1}{2} (G^*(z) + G(z)), \quad F_\sigma(z) = \frac{1}{2iz} (G^*(z) - G(z)),
$$

where the operation $^*$ is defined as $f^*(z) = \overline{f(\overline{z})}$. These entire functions belong to the Cartwright class $\mathcal{C}$, and their types are equal to $\sigma$. These functions are of strict least deviation from zero among the entire functions of the class $\mathcal{C}$ relative to the uniform metric on $\mathbb{R}$ with the weights $1/\rho_m$ and $| \cdot |/\rho_m$, respectively. In particular, it is proved, if a function $\varphi : \mathbb{R} \to \mathbb{R}$ is even and convex, and if $\varphi(0) = 0$, $\varphi(u) > 0$ for $u \neq 0$, and $\varphi(2u) \leq K \varphi(u)$ for $u \geq u_0$, then for any entire function $k_\alpha$ of type $\alpha \leq \sigma$ such that $k_\alpha/\rho_m \in L_1$,

$$
\int_{-\infty}^{\infty} \varphi' \left(\frac{f_\sigma}{\rho_m}\right) \frac{k_\alpha}{\rho_m} = 0.
$$

Similar results are obtained for the functions $F_\sigma(z)$. Earlier, the authors constructed functions least deviating from zero among the entire functions of type $\sigma$ in the uniform and integral metrics on $\mathbb{R}$ with the weights $\omega = 1/\rho_m$ and $\omega = | \cdot |/\rho_m$. In this paper it is shown that these functions deviate least from zero in some other classes which are related to the function $\rho_m$ and generalize the Orlicz classes. In particular, the results are obtained for the spaces $L_p(\mathbb{R})$, $p < \infty$, with the weight $\omega^p$ for the same $\omega$.

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MSC:
30D15 Special classes of entire functions of one complex variable and growth estimates
41A50 Best approximation, Chebyshev systems
30D60 Quasi-analytic and other classes of functions of one complex variable

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References:
