Kurakin, Leonid G.; Lysenko, Irina A.; Ostrovskaya, Irina V.; Sokolovskiy, Mikhail A.

On stability of the Thomson's vortex N-gon in the geostrophic model of the point vortices in two-layer fluid. (English) Zbl 1423.76076

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Summary: A two-layer quasigeostrophic model is considered. The stability analysis of the stationary rotation of a system of N identical point vortices lying uniformly on a circle of radius R in one of the layers is presented. The vortices have identical intensity and length scale is $\gamma^{-1} > 0$. The problem has three parameters: $N, \gamma R$ and β, where β is the ratio of the fluid layer thicknesses. The stability of the stationary rotation is interpreted as orbital stability. The instability of the stationary rotation is instability of system reduced equilibrium. The quadratic part of the Hamiltonian and eigenvalues of the linearization matrix are studied. The parameter space $(N, \gamma R, \beta)$ is divided on three parts: $\boldsymbol{A}$ is the domain of stability in an exact nonlinear setting, $\boldsymbol{B}$ is the linear stability domain, where the stability problem requires the nonlinear analysis, and $\boldsymbol{C}$ is the instability domain. The case $\boldsymbol{A}$ takes place for $N = 2, 3, 4$ for all possible values of parameters γR and β. In the case of $N = 5$, we have two domains: $\boldsymbol{A}$ and $\boldsymbol{B}$. In the case $N = 6$, part $\boldsymbol{B}$ is curve, which divides the space of parameters $(\gamma R, \beta)$ into the domains: $\boldsymbol{A}$ and $\boldsymbol{C}$. In the case of $N = 7$, there are all three domains: $\boldsymbol{A}, \boldsymbol{B}$ and $\boldsymbol{C}$. The instability domain $\boldsymbol{C}$ takes place always if $N = 2n \geqslant 8$. In the case of $N = 2\ell + 1 \geqslant 9$, there are two domains: $\boldsymbol{B}$ and $\boldsymbol{C}$. The results of research are presented in two versions: for parameter β and parameter α, where α is the difference between layer thicknesses. A number of statements about the stability of the Thomson N-gon is obtained for the systems of interacting particles with the general Hamiltonian depending only on distances between the particles. The results of theoretical analysis are confirmed by numerical calculations of the vortex trajectories.

MSC:

76B47 Vortex flows for incompressible inviscid fluids
76E20 Stability and instability of geophysical and astrophysical flows
34D20 Stability of solutions to ordinary differential equations

Keywords:

N-vortex problem; point vortices; two-layer fluid; Hamiltonian dynamics; stability

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References:

[1] Abramowitz, M., Stegun, I.A. (eds.): Handbook of Mathematical Functions with Formulas, Graphs and Mathematical Tables. Applied Mathematics Series, vol. 55. National Bureau of standards, Gaithersburg (1964) · Zbl 0171.38503

[2] Aref, H.; Newton, PK; Stremler, MA; Tokieda, T.; Vainchtein, DL, Vortex crystals, Adv. Appl. Mech., 39, 1-79, (2003)· doi:10.1016/S0065-2156(02)39001-X

[3] Arnold, VI, Small denominators and problems of stability of motion in classical and celestial mechanics, Russ. Math. Surv., 18, 85-191, (1963) · Zbl 0135.42701 · doi:10.1070/RM1963v018n06ABEH001143

[4] Borisov, A.V., Mamaev, I.S.: Mathematical Methods in the Dynamics of Vortex Structures. Institute of Computer Sciences, Moscow (2005) · Zbl 1119.76001

[5] Gantmacher, F.R.: Lectures on Analytical Mechanics. Glavnaya Redaktsiya Fiziko-Matematicheskoj Literatury, Nauka, Moscow (1966)

[6] Gryanik, VM, Dynamics of singular geostrophic vortices in a two-level model of atmosphere (ocean), Izv. Atmos. Ocean. Phys., 19, 227-240, (1983)

[7] Havelock, TH, The stability of motion of rectilinear vortices in ring formation, Philos. Mag., 11, 617-633, (1931) · Zbl 57.1109.03 · doi:10.1080/14786443109461714

[8] Karapetyan, AV, The Routh Theorem and Its Extensions, No. 53, 271-290, (1990), Amsterdam

[9] Kelvin, W.T.: Floating Magnets (Illustrating Vortex Systems). Mathematical and Physical Papers, vol. 4, pp. 162-164. Cambridge University press, Cambridge (1910)

[10] Kolmogorov, AN, On conservation of conditionally periodic motions for a small change in Hamilton's function, Dokl. Akad. Nauk SSSR, 98, 527-530, (1954) · Zbl 0056.31502

[11] Kurakin, LG, Stability, resonances, and instability of the regular vortex polygons in the circular domain, Dokl. Phys., 49, 658-661, (2004)· doi:10.1134/1.1831532

[12] Kurakin, LG; Ostrovskaya, IV, Nonlinear stability analysis of a regular vortex pentagon outside a circle, Regul. Chaotic Dyn., 17, 385-396, (2012) · Zbl 1252.76017 · doi:10.1134/S1560354712050024

[13] Kurakin, LG; Ostrovskaya, IV, On stability of the Thomson's vortex \(N\)-gon in the geostrophic model of the point Bessel vortices, Regul. Chaotic Dyn., 22, 865-879, (2017) · Zbl 1401.76037 · doi:10.1134/S1560354717070085

[14] Kurakin, LG; Yudovich, VI, The stability of stationary rotation of a regular vortex polygon, Chaos, 12, 574-595, (2002) · Zbl 1080.76520 · doi:10.1063/1.1482175

[15] Kurakin, LG; Yudovich, VI, On nonlinear stability of steady rotation of a regular vortex polygon, Dokl. Phys., 47, 465-470, (2002) · Zbl 1080.76520 · doi:10.1134/1.1493390

[16] Kurakin, LG; Ostrovskaya, IV; Sokolovskiy, MA, On the stability of discrete tripole, quadrupole, Thomson' vortex triangle and square in a two-layer/homogeneous rotating fluid, Regul. Chaotic Dyn., 21, 291-334, (2016) · Zbl 1346.76204 · doi:10.1134/S1560354716030059

[17] Kurakin, LG; Melekhov, AP; Ostrovskaya, IV, A survey of the stability criteria of Thomson's vortex polygons outside a circular domain, Bol. Soc. Mat. Mex., 22, 733-744, (2016) · Zbl 1348.76041 · doi:10.1007/s40590-016-0121-y

[18] Markeev, A.P.: Libration Points in Celestial Mechanics and Space Dynamics. Nauka, Moscow (1978)

[19] Morikawa, GK; Swenson, EV, Interacting motion of rectilinear geostrophic vortices, Phys. Fluids, 14, 1058-1073, (1971)· doi:10.1063/1.1693564

[20] Moser, J.: Lectures on Hamiltonian Systems, vol. 81. Memoirs of the American Mathematical Society, Providence (1968) · Zbl 0172.11401

[21] Newton, P.K.: The N-Vortex Problem: Analytical Techniques. Applied Mathematical Sciences, vol. 145. Springer, New York (2001) · Zbl 0981.76002 · doi:10.1007/978-1-4684-9290-3

[22] Proskuryakov, I.V.: A Collection of Problems in Linear Algebra. Textbook (Sbornik zadach po linejnoj algebre. Uchebnoe posobie), 7th ed. Moscow, Glavnaya Redaktsiya Fiziko-Matematicheskoj Literatury, Nauka (1984) · Zbl 0544.15001

[23] Routh, E.J.: A Treatise on the Stability of a Given State Motion. Macmillan, London (1877)

[24] Saffman, P.G.: Vortex Dynamics. Cambridge Monographs on Mechanics and Applied Mathematics. Cambridge University Press, Cambridge (1992) · Zbl 0777.76004

[25] Sokolovskiy, M.A., Verron, J.: Dynamics of Vortex Structures in a Stratified Rotating Fluid. Atmospheric and Oceanographic Sciences Library, vol. 47. Springer, Cham (2014) · Zbl 1384.86001 · doi:10.1007/978-3-319-00789-2

[26] Stewart, HJ, Periodic properties of the semi-permanent atmospheric pressure systems, Q. Appl. Math., 1, 262, (1943) · Zbl 0063.07196 · doi:10.1090/qam/9349

[27] Stewart, HJ, Hydrodynamic problems arising from the investigation of the transverse circulation in the atmosphere, Bull. Am. Math. Soc., 51, 781-799, (1945) · Zbl 0063.07197 · doi:10.1090/S0002-9904-1945-08443-2

[28] Thomson, W., Floating magnets (illustrating vortex systems), Nature, 18, 13-14, (1878)· doi:10.1038/018013b0

[29] Thomson, J.J.: A Treatise on the Motion of Vortex Rings, pp. 94-108. Macmillan, London (1883)