Czyżycki, Tomasz; Hrivnák, Jiří; Patera, Jiří
Generating functions for orthogonal polynomials of $A_2$, $C_2$ and $G_2$. (English) Zbl 1423.33023

Summary: The generating functions of fourteen families of generalized Chebyshev polynomials related to rank two Lie algebras $A_2$, $C_2$ and $G_2$ are explicitly developed. There exist two classes of the orthogonal polynomials corresponding to the symmetric and antisymmetric orbit functions of each rank two algebra. The Lie algebras $G_2$ and $C_2$ admit two additional polynomial collections arising from their hybrid character functions. The admissible shift of the weight lattice permits the construction of a further four shifted polynomial classes of $C_2$ and directly generalizes formation of the classical univariate Chebyshev polynomials of the third and fourth kinds. Explicit evaluating formulas for each polynomial family are derived and linked to the incomplete exponential Bell polynomials.

MSC:
33C80 Connections of hypergeometric functions with groups and algebras, and related topics
33C45 Orthogonal polynomials and functions of hypergeometric type (Jacobi, Laguerre, Hermite, Askey scheme, etc.)

Keywords:
generating function; root system; orthogonal polynomial; Weyl group; Lie algebra

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