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Description of closure operators in convex geometries of segments on the line. (English)

Zbl 1423.05003
Algebra Univers. 80, No. 4, Paper No. 44, 22 p. (2019).

Summary: Convex geometry is a closure space (G,ϕ) with the anti-exchange property. A classical result of *P. H. Edelman* and *R. E. Jamison* [Geom. Dedicata 19, 247–270 (1985; Zbl 0577.52001)] claims that every finite convex geometry is a join of several linear sub-geometries, and the smallest number of such sub-geometries necessary for representation is called the convex dimension. In our work we find necessary and sufficient conditions on a closure operator ϕ of convex geometry (G,ϕ) so that its convex dimension equals 2, equivalently, they are represented by segments on a line. These conditions, for a given convex geometry (G,ϕ), can be checked in polynomial time in two parameters: the size of the base set $|G|$ and the size of the implicational basis of (G,ϕ).

MSC:

05A05 Permutations, words, matrices
06A15 Galois correspondences, closure operators (in relation to ordered sets)
06B99 Lattices
52B55 Computational aspects related to convexity

Keywords:

closure system; convex geometry; anti-exchange property; affine convex geometry; implicational basis; convex dimension; extreme points; Carathéodory condition; convex geometry of circles; convex geometry of segments

Full Text:

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