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On residuated lattices with left and right internal state. (English) Zbl 1423.03259

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Summary: In this paper, notions of left- and right-state operators on residuated lattices are introduced and some related properties of such operators are investigated. Filters and normal filters generated by a subset in a state residuated lattice are characterized and it is shown that the lattice of filters forms a frame. Subdirectly irreducible state residuated lattices are characterized. The notion of state coannihilator is introduced and a connection between them and Galois connection is established. Finally, it is shown that the set of state coannihilators forms a complete Boolean algebra.

MSC:

03G25 Other algebras related to logic
06F05 Ordered semigroups and monoids
06A15 Galois correspondences, closure operators (in relation to ordered sets)
06D20 Heyting algebras (lattice-theoretic aspects)

Keywords:

residuated lattice; state residuated lattice; Galois connection; state filter; state congruence; Heyting algebra; state coannihilator

Full Text: DOI

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