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**A new definition of order relation for the introduction of algebraic fuzzy closure operators.**

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Summary: In this paper, a new approach to order relation between fuzzy sets is provided, which is called well inclusion order between fuzzy sets. Based on this new order relation, the concept of algebraic fuzzy closure operators is introduced. It is shown that there is a categorical isomorphism between algebraic fuzzy closure operators and fuzzy convex structures. Also, the relationship between fuzzy closure systems and fuzzy convex structures is investigated. It is proved that the category of fuzzy convex spaces is a bicoreflective subcategory of the category of fuzzy closure system spaces.

**MSC:**

03E72 Theory of fuzzy sets, etc.
06A15 Galois correspondences, closure operators (in relation to ordered sets)
03G25 Other algebras related to logic



**Keywords:**

fuzzy set; fuzzy closure system; fuzzy closure operator; fuzzy convex structure

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