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Self-excited oscillations in a collapsible channel with applications to retinal venous pulsation. (English) Zbl 1422.76213

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Summary: We consider a theoretical model for the flow of Newtonian fluid through a long flexible-walled channel which is formed from four compliant and rigid compartments arranged alternately in series. We drive the flow using a fixed upstream flux and derive a spatially one-dimensional model using a flow profile assumption. The compliant compartments of the channel are assumed subject to a large external pressure, so the system admits a highly collapsed steady state. Using both a global (linear) stability eigensolver and fully nonlinear simulations, we show that these highly collapsed steady states admit a primary global oscillatory instability similar to observations in a single channel. We also show that in some regions of the parameter space the system admits a secondary mode of instability which can interact with the primary mode and lead to significant changes in the structure of the neutral stability curves. Finally, we apply the predictions of this model to the flow of blood through the central retinal vein and examine the conditions required for the onset of self-excited oscillation. We show that the neutral stability curve of the primary mode of instability discussed above agrees well with canine experimental measurements of the onset of retinal venous pulsation, although there is a large discrepancy in the oscillation frequency.

MSC:

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