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Three-dimensional small-scale instabilities of plane internal gravity waves. (English)

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Summary: We study the evolution of three-dimensional (3-D), small-scale, small-amplitude perturbations on a plane internal gravity wave using the local stability approach. The plane internal wave is characterised by its non-dimensional amplitude, A, and the angle the group velocity vector makes with gravity, Φ. For a given (A,Φ), the local stability equations are solved on the periodic fluid particle trajectories to obtain growth rates for all two-dimensional (2-D) and 3-D perturbation wave vectors. For small A, the local stability approach recovers previous results of 2-D parametric subharmonic instability (PSI) while offering new insights into 3-D PSI. Higher-order triadic resonances, and associated deviations from them, are also observed at small A. Moreover, for small A, purely transverse instabilities resulting from parametric resonance are shown to occur at select values of Φ. The possibility of a non-resonant instability mechanism for transverse perturbations at finite A allows us to derive a heuristic, modified gravitational instability criterion. We then study the extension of small A to finite A internal wave instabilities, where we recover and build upon existing knowledge of small-scale, small-amplitude internal wave instabilities. Four distinct regions of the (A,Φ)-plane based on the dominant instability modes are identified: 2-D PSI, 3-D oblique, quasi-2-D shear-aligned, and 3-D transverse. Our study demonstrates the local stability approach as a physically insightful and computationally efficient tool, with potentially broad utility for studies that are based on other theoretical approaches and numerical simulations of small-scale instabilities of internal waves in various settings.

MSC:

86A05 Hydrology, hydrography, oceanography
86A10 Meteorology and atmospheric physics
76E20 Stability and instability of geophysical and astrophysical flows
76B15 Water waves, gravity waves; dispersion and scattering, nonlinear interaction

Keywords:

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