Allwright, Emma J.; **Forbes, L. K.**; **Walters, S. J.**
Axisymmetric plumes in viscous fluids. (English) Zbl 1415.76243
ANZIAM J. 61, No. 2, 119-147 (2019).

Summary: We consider fluid in a channel of finite height. There is a circular hole in the channel bottom, through which fluid of a lower density is injected and rises to form a plume. Viscous boundary layers close to the top and bottom of the channel are assumed to be so thin that the viscous fluid effectively slips along each of these boundaries. The problem is solved using a novel spectral method, in which Hankel transforms are first used to create a steady-state axisymmetric (inviscid) background flow that exactly satisfies the boundary conditions. A viscous correction is then added, so as to satisfy the time-dependent Boussinesq Navier-Stokes equations within the fluid, leaving the boundary conditions intact. Results are presented for the "lazy" plume, in which the fluid rises due only to its own buoyancy, and we study in detail its evolution with time to form an overturning structure. Some results for momentum-driven plumes are also presented, and the effect of the upper wall of the channel on the evolution of the axisymmetric plume is discussed.

MSC:

76E20 Stability and instability of geophysical and astrophysical flows
76E17 Interfacial stability and instability in hydrodynamic stability
76M22 Spectral methods applied to problems in fluid mechanics

Keywords:

Boussinesq fluid; overturning plume; unsteady flow

Full Text: DOI

References:

[1] Abramowitz, M.; Stegun, I. A., Handbook of mathematical functions, (1972), Dover: Dover, New York · Zbl 0543.33001

[2] Andrews, M. J.; Dalziel, S. B., Small Atwood number Rayleigh-Taylor experiments, Philos. Trans. R. Soc. Lond. Ser. A, 368, 1663-1679, (2010) · Zbl 1192.76004

[3] Atkinson, K. E., Elementary numerical analysis, (1985), Wiley: Wiley, New York · Zbl 0645.65002

[4] Baker, G.; Caflisch, R. E.; Siegel, M., Singularity formation during Rayleigh-Taylor instability, J. Fluid Mech., 252, 51-78, (1993) · Zbl 0791.76027

[5] Baker, G. R.; Pham, L. D., A comparison of blob methods for vortex sheet roll-up, J. Fluid Mech., 547, 297-316, (2006) · Zbl 1082.76078

[6] Batchelor, G. K., An introduction to fluid dynamics, (1967), Cambridge University Press: Cambridge University Press, Cambridge · Zbl 0152.44402

[7] Boyd, J. P., Chebyshev and Fourier spectral methods, (2001), Dover: Dover, New York · Zbl 0994.65128

[8] Craske, J.; van Reeuwijk, M., Generalised unsteady plume theory, J. Fluid Mech., 792, 1013-1052, (2016) · Zbl 1381.76125

[9] Farrow, D. E.; Hocking, G. C., A numerical model for withdrawal from a two-layer fluid, J. Fluid Mech., 549, 141-157, (2006)

[10] Forbes, L. K., The Rayleigh-Taylor instability for inviscid and viscous fluids, J. Engrg. Math., 65, 273-290, (2009) · Zbl 1180.76023

[11] Forbes, L. K., A cylindrical Rayleigh-Taylor instability: radial outflow from pipes or stars, J. Engrg. Math., 70, 205-224, (2011) · Zbl 1254.76084

[12] Forbes, L. K., Rayleigh-Taylor instabilities in axi-symmetric outflow from a point source, ANZIAM J., 53, 87-121, (2011) · Zbl 1247.76032

[13] Forbes, L. K., How strain and spin may make a star bipolar, J. Fluid Mech., 746, 332-367, (2014) · Zbl 1309.85004

[14] Gómez, L.; Rodríguez, L. F.; Loinard, L., A one-sided knot ejection at the core of the HH 111 outflow, Rev. Mex. Astron. Astrofís., 49, 79-85, (2013)

[15] Gradshteyn, I. S.; Ryzhik, I. M., Tables of integrals, series and products, (2000), Academic Press: Academic Press, San Diego, CA · Zbl 0981.65001

[16] Hocking, G. C.; Forbes, L. K., Steady flow of a buoyant plume into a constant-density layer, J. Engrg. Math., 67, 341-350, (2010) · Zbl 1310.76176

[17] Horsley, D. E., Bessel phase functions: calculation and application, Numer. Math., 136, 679-702, (2017) · Zbl 1456.33007

[18] Hunt, G. R.; Burridge, H. C., Fountains in industry and nature, Annu. Rev. Fluid Mech., 47, 195-220, (2015)

[19] Hunt, G. R.; Kaye, N. B., Lazy plumes, J. Fluid Mech., 533, 329-338, (2005) · Zbl 1074.76057

[20] Krasny, R., Desingularization of periodic vortex sheet roll-up, J. Comput. Phys., 65, 292-313, (1986) · Zbl 0591.76059

[21] Letchford, N. A.; Forbes, L. K.; Hocking, G. C., Inviscid and viscous models of axisymmetric fluid jets or plumes, ANZIAM J., 53, 228-250, (2012) · Zbl 1254.76074

[22] Lovelace, R. V. E.; Romanova, M. M.; Ustyugova, G. V.; Koldoba, A. V., One-sided outflows/jets from rotating stars with complex magnetic fields, Mon. Not. R. Astron. Soc., 408, 2083-2091, (2010)

[23] Matsuoka, C.; Nishihara, K., Analytical and numerical study on a vortex sheet in incompressible Richtmyer-Meshkov instability in cylindrical geometry, Phys. Rev. E, 74, (2006)

[24] Mikaelian, K. O., Rayleigh-Taylor and Richtmyer-Meshkov instabilities and mixing in stratified cylindrical shells, Phys. Fluids, 17, (2005) · Zbl 1187.76353

[25] Moore, D. W., The spontaneous appearance of a singularity in the shape of an evolving vortex sheet, Proc. R. Soc. Lond. Ser. A, 365, 105-119, (1979) · Zbl 0404.76040

[26] Morton, B. R.; Taylor, G. I.; Turner, J. S., Turbulent gravitational convection from maintained and instantaneous sources, Proc. R. Soc. Lond. Ser. A, 234, 1-23, (1956) · Zbl 0074.45402

[27] Proskurowski, G.; Lilley, M. D.; Kelley, D. S.; Olson, E. J., Low temperature volatile production at the Lost City Hydrothermal Field, evidence from a hydrogen stable isotope geothermometer, Chem. Geol., 229, 331-343, (2006)

[28] Rayleigh, Lord, Investigation of the character of the equilibrium of an incompressible heavy fluid of variable density, Proc. Lond. Math. Soc. (3), 14, 170-177, (1883) · Zbl 15.0848.02

[29] Russell, P. S.; Forbes, L. K.; Hocking, G. C., The initiation of a planar fluid plume beneath a rigid lid, J. Engrg. Math., 106, 107-121, (2017) · Zbl 1469.76039

[30] Sharp, D. H., An overview of Rayleigh-Taylor instability, Physica D, 12, 3-18, (1984) · Zbl 0577.76047

[31] Stahler, S. W.; Palla, F., The formation of stars, (2004), Wiley-VCH: Wiley-VCH, Berlin

[32] Taylor, Sir G. I., The instability of liquid surfaces when accelerated in a direction perpendicular to their planes, I, Proc. R. Soc. Lond. Ser. A, 201, 192-196, (1950) · Zbl 0038.12201

[33] Trefethen, L. N.

[34] Vadivukkarasan, M.; Panchagnula, M. V., Combined Rayleigh-Taylor and Kelvin-Helmholtz instabilities on an annular liquid sheet, J. Fluid Mech., 812, 152-177, (2017) · Zbl 1383.76148

[35] von Winckel, G.

[36] Woods, A. W., Turbulent plumes in nature, Annu. Rev. Fluid Mech., 42, 391-412, (2010)