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Uniform approximation to finite Hilbert transform of oscillatory functions and its algorithm.

Summary: For the finite Hilbert transform of oscillatory functions
\[ Q(f; c, \omega) = -\int_{-1}^{1} f(x) e^{i\omega x} / (x - c) \, dt \]
with a smooth function \( f \) and real \( \omega \neq 0 \), for \( c \in (-1, 1) \) in the sense of Cauchy principal value or for \( c = \pm 1 \) of Hadamard finite-part, we present an approximation method of Clenshaw-Curtis type and its algorithm. Interpolating \( f \) by a polynomial \( p_n \) of degree \( n \) and expanding in terms of the Chebyshev polynomials with \( O(n \log n) \) operations by the FFT, we obtain an approximation \( Q(p_n; c, \omega) \approx Q(f; c, \omega) \).

We write \( Q(p_n; c, \omega) \) as a sum of the sine and cosine integrals and an oscillatory integral of a polynomial of degree \( n - 1 \). We efficiently evaluate the oscillatory integral with a combination of authors’ previous method and Keller’s method. For \( f(z) \) analytic on the interval \([-1, 1]\) in the complex plane \( z \), the error of \( Q(p_n; c, \omega) \) is bounded uniformly with respect to \( c \) and \( \omega \). Numerical examples illustrate the performance of our method.

MSC:
65D32 Numerical quadrature and cubature formulas
41A55 Approximate quadratures
65T40 Numerical methods for trigonometric approximation and interpolation

Keywords:
quadrature rule; principal value integral; oscillatory function; Chebyshev interpolation; error analysis; uniform approximation

Software:
Clenshaw-Curtis; mctoolbox

Full Text: DOI

References: