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Pricing and risk of swing contracts in natural gas markets. (English) Zbl 1414.91380
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Summary: Motivated by the growing importance of swing contracts in natural gas markets, this article extends the literature on commodity price modelling as well as valuation methods and sensitivity analysis for swing options. While most previous studies focused on simple price models, we face the challenge of deriving option properties under more realistic commodity price dynamics. We begin by formulating a multi-factor price forward curve model with parametric volatility functions, which can capture uncertainty in both yearly seasonality and time-to-maturity effects, and propose a two-step calibration procedure to fit such models to empirical data. We then show how results from the literature can be combined to obtain swing option values and sensitivities in such a general framework. In this context, we also provide new theoretical results and a first numerical approach to efficiently estimate swing options' gammas. For options' deltas, we expand upon existing studies by including a larger variety of contract specifications and by focusing on a multidimensional variant of the Longstaff-Schwartz algorithm as an alternative option valuation method. With these contributions, we supply important tools for swing option sellers and buyers relying on accurate option value and risk estimates to maintain their business models, hedge option-related risks and adequately represent swing options in financial reporting.

MSC:

91G20 Derivative securities (option pricing, hedging, etc.)
91G60 Numerical methods (including Monte Carlo methods)

Keywords:

natural gas; forward curve dynamics; swing options; delta; gamma; simulation

Software:

bootstrap

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