

**Zhong, Xin**

**Strong solutions to the 2D Cauchy problem of density-dependent viscous Boussinesq equations with vacuum.** (English) [Zbl 1414.76021](#)

*J. Math. Phys.* 60, No. 5, 051505, 15 p. (2019).

**Summary:** This paper concerns the Cauchy problem of the density-dependent Boussinesq equations without dissipation term on the temperature equation on the whole space  $\mathbb{R}^2$  with vacuum as far field density. We show that there exists a unique local strong solution provided the initial density and the initial temperature decay not too slow at infinity. In particular, the initial data can be arbitrarily large and the initial density may contain vacuum states. Moreover, there is no need to require any Cho-Choe-Kim type compatibility conditions.

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**MSC:**

**76E06** Convection in hydrodynamic stability

**76N10** Existence, uniqueness, and regularity theory for compressible fluids and gas dynamics

**35F10** Initial value problems for linear first-order PDEs

**76E20** Stability and instability of geophysical and astrophysical flows

**86A10** Meteorology and atmospheric physics

**35Q35** PDEs in connection with fluid mechanics

Cited in **2** Documents

**Full Text:** [DOI](#)

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