Henry, D.; **Martin, C. I.**

Free-surface, purely azimuthal equatorial flows in spherical coordinates with stratification. (English) Zbl 1412.35241

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Summary: In this paper we derive an exact solution to the governing equations for geophysical fluid dynamics in spherical coordinates which incorporates fluid stratification. This solution represents a steady, purely-azimuthal equatorial flow with an associated free-surface. Following the derivation of the solution we demonstrate that there is a well-defined relationship between the imposed pressure at the free-surface and the resulting distortion of the surface's shape. Finally, the solution for stratified fluid flow is subjected to a short-wavelength stability analysis.

MSC:

35Q31 Euler equations
35Q35 PDEs in connection with fluid mechanics
35Q86 PDEs in connection with geophysics
35R35 Free boundary problems for PDEs
76E20 Stability and instability of geophysical and astrophysical flows
76U05 General theory of rotating fluids
86A05 Hydrology, hydrography, oceanography

Keywords:

azimuthal flows; variable (linear) density; spherical coordinates; Coriolis force; implicit function theorem; short-wavelength perturbation

Full Text: DOI

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