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Instability of nonlinear wave-current interactions in a modified equatorial β-plane approximation. (English) Zbl 1411.76015

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Summary: We present an instability analysis of some exact and explicit solutions to the geophysical equatorial β-plane equations incorporating a gravitational-correction term. A criterion for the instability is given by means of the short-wavelength perturbation method. Thresholds for both, a solution with a zonal current under constant density and a solution admitting stratification, are derived and expressed in terms of the steepness of the waves.

MSC:

76B15	Water waves, gravity waves; dispersion and scattering, nonlinear interaction	Cited in **2** Documents
76E20	Stability and instability of geophysical and astrophysical flows	
37H15	Random dynamical systems aspects of multiplicative ergodic theory, Lyapunov exponents	

Keywords:

geophysical flows; exact solutions; short-wavelength stability method

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