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Hindrances to bistable front propagation: application to *Wolbachia* invasion. (English)

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The authors study the biological situation when an invading population propagates and replaces an existing population with different characteristics. They aim at quantifying the propagules and the invasive power. They rigorously show that a heterogeneous environment inducing a strong enough population gradient can stop an invading front, which will converge in this case to a stable front. They characterize the critical population jump, and also prove the existence of unstable fronts above the stable (blocking) fronts. They are particularly interested in the case of artificial Wolbachia infection, used as a tool to fight arboviruses.

The main results are the characterization of the asymptotic behavior of p in two settings: when N (total population density) only depends on x and when $\partial x \log N$ is equal to a constant time the characteristic function of an interval. Overall, two possible sets of asymptotic behaviors appear. Also they demand a numerical conjecture that for generic bistable function, there exists exactly two barriers.

Reviewer: Abdolrahman Razani (Qazvin)

MSC:

34B18	Positive solutions to nonlinear boundary value problems for ordinary differential equations	Cited in **3** Documents
35K57	Reaction-diffusion equations	
35B40	Asymptotic behavior of solutions to PDEs	
92D25	Population dynamics (general)	
35C07	Traveling wave solutions	

Keywords:

front propagation; wave-blocking; bistable reaction-diffusion; shooting argument; Wolbachia

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