

Dritschel, David Gerard; Hmidi, Taoufik; Renault, Coralie

Imperfect bifurcation for the quasi-geostrophic shallow-water equations. (English)

Zbl 1409.76036

Arch. Ration. Mech. Anal. 231, No. 3, 1853-1915 (2019).

This is a piece of nice analytical and numerical work. Fluid by nature is ever changing, as a result, the fluid equations usually display turbulent solutions. Stationary or steady solutions of fluid equations are usually trivial. The stationary uniform vorticity patches are interesting steady solutions, which can have various applications. This work demonstrates bifurcation patterns near these stationary vortex patches based on the Crandall-Rabinowitz theorem. Numerical simulation of the problem is also challenging, and the current work pushes the numerical simulation to an impressive stage.

Reviewer: [Y. Charles Li \(Columbia\)](#)

MSC:

76E20 Stability and instability of geophysical and astrophysical flows

76M99 Basic methods in fluid mechanics

Cited in **6** Documents

Keywords:

uniform vorticity; Crandall-Rabinowitz theorem; vortex patch

Full Text: [DOI](#) [arXiv Link](#)

References:

- [1] Burbea, J., Motions of vortex patches, *Lett. Math. Phys.*, 6, 1-16, (1982) · [Zbl 0484.76031](#)
- [2] Castro, A.; Córdoba, D.; Gomez-Serrano, J., Existence and regularity of rotating global solutions for the generalized surface quasi-geostrophic equations, *Duke Math. J.*, 165, 935-984, (2016) · [Zbl 1339.35234](#)
- [3] Castro, A., Córdoba, D., Gomez-Serrano, J.: *Uniformly rotating analytic global patch solutions for active scalars*. *Ann. PDE* **2**(1), Art. 1, 34 2016 · [Zbl 1397.35020](#)
- [4] Cerretelli, C.; Williamson C. H., K., A new family of uniform vortices related to vortex configurations before merging, *J. Fluid Mech.*, 493, 219-229, (2003) · [Zbl 1063.76008](#)
- [5] Crandall, M. G.; Rabinowitz, P. H., Bifurcation from simple eigenvalues, *J. Funct. Analysis*, 8, 321-340, (1971) · [Zbl 0219.46015](#)
- [6] Deem, G. S.; Zabusky, N. J., Vortex waves: stationary “V-states”, interactions, recurrence, and breaking, *Phys. Rev. Lett.*, 40, 859-862, (1978)
- [7] Dritschel, D. G., The nonlinear evolution of rotating configurations of uniform vorticity, *J. Fluid Mech.*, 172, 157-172, (1986) · [Zbl 0616.76069](#)
- [8] Dritschel, D. G., Contour surgery: a topological reconnection scheme for extended integrations using contour dynamics, *J. Comput. Phys.*, 77, 240-266, (1988) · [Zbl 0642.76025](#)
- [9] Dritschel, D. G., Contour dynamics and contour surgery: numerical algorithms for extended, high-resolution modelling of vortex dynamics in two-dimensional, inviscid, incompressible flows, *Computer Phys. Rep.*, 10, 77-146, (1989)
- [10] Dritschel, D. G., A general theory for two-dimensional vortex interactions, *J. Fluid Mech.*, 293, 269-303, (1995) · [Zbl 0854.76018](#)
- [11] Golubitsky, M.; Schaeffer, D., A theory for imperfect bifurcation via singularity theory, *Commun. Pure Appl. Math.*, 32, 21-98, (1979) · [Zbl 0409.58007](#)
- [12] Hassainia, Z.; Hmidi, T., On the V-states for the generalized quasi-geostrophic equations, *Commun. Math. Phys.*, 337, 321-377, (2015) · [Zbl 1319.35188](#)
- [13] Hassainia, Z., Masmoudi, N., Wheeler, M. H.: *Global bifurcation of rotating vortex patches*, [arXiv:1712.03085](#)
- [14] Hmidi, T.; Mateu, J.; Verdera, J., Boundary regularity of rotating vortex patches, *Arch. Ration. Mech. Anal.*, 209, 171-208, (2013) · [Zbl 1286.35201](#)
- [15] Hmidi, T.; Mateu, J., Bifurcation of rotating patches from Kirchhoff vortices, *Discrete Contin. Dyn. Syst.*, 36, 5401-5422, (2016) · [Zbl 1351.35116](#)
- [16] Kamm, J. R.: *Shape and stability of two-dimensional uniform vorticity regions*. Ph.D. thesis, California Institute of Technology, 1987
- [17] Kirchhoff G.R.: *Vorlesungen über mathematische Physik. Mechanik*. Teubner, Leipzig (1876) · [Zbl 08.0542.01](#)

- [18] Liu, P.; Shi, J.; Wang, Y., Imperfect transcritical and pitchfork bifurcations, *J. Funct. Anal.*, 251, 573-600, (2007) · [Zbl 1139.47042](#)
- [19] Love, A. E.H., On the stability of certain vortex motions, *Proc. Lond. Math. Soc.*, 35, 18, (1893) · [Zbl 25.1467.02](#)
- [20] Luzzatto-Fegiz, P.; Williamson, C. H.K., Stability of elliptical vortices from “Imperfect-Velocity-Impulse” diagrams, *Theor. Comput. Fluid Dyn.*, 24, 181-188, (2010) · [Zbl 1191.76056](#)
- [21] Luzzatto-Fegiz, P.; Williamson, C. H.K., An efficient and general numerical method to compute steady uniform vortices, *J. Comput. Phys.*, 230, 6495-6511, (2011) · [Zbl 1408.76123](#)
- [22] Overman, E. A., Steady-state solutions of the Euler Equations in two dimensions II Local analysis of limiting V-states., *SIAM J. Appl. Math.*, 46, 765-800, (1986) · [Zbl 0608.76018](#)
- [23] PŁotka, H.; Dritschel, D. G., Quasi-geostrophic shallow-water vortex-patch equilibria and their stability, *Geophys. Astrophys. Fluid Dyn.*, 106, 574-595, (2012)
- [24] Polvani, L. M.: \textit{Geostrophic vortex dynamics}. PhD thesis, MIT/WHOI WHOI-88-48, 1988
- [25] Polvani, L. M.; Zabusky, N. J.; Flierl, G. R., Two-layer geostrophic vortex dynamics. Part 1 Upper-layer V-states and merger., *J. Fluid Mech.*, 205, 215-242, (1989) · [Zbl 0676.76093](#)
- [26] Segura, J., Bounds for ratios of modified Bessel functions and associated Turán-type inequalities, *J. Math. Anal. Appl.*, 374, 516-528, (2011) · [Zbl 1207.33009](#)
- [27] Shi, J., Persistence and bifurcation of degenerate solutions, *J. Funct. Anal.*, 169, 494-531, (1999) · [Zbl 0949.47050](#)
- [28] Vallis G.K.: *Atmospheric and Oceanic Fluid Dynamics*. Cambridge University Press, Cambridge (2008) · [Zbl 1374.86002](#)
- [29] Watson G.N.: *A Treatise on the Theory of Bessel Functions*. Cambridge University Press, Cambridge (1944) · [Zbl 0063.08184](#)

This reference list is based on information provided by the publisher or from digital mathematics libraries. Its items are heuristically matched to zbMATH identifiers and may contain data conversion errors. It attempts to reflect the references listed in the original paper as accurately as possible without claiming the completeness or perfect precision of the matching.